Universidade da Coruña
Departamento de Computación

New Compression Codes for Text Databases

Tese Doutoral

Doutorando: Antonio Fariña Martínez
Directores: Nieves R. Brisaboa e Gonzalo Navarro

A Coruña, Abril de 2005

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Aos meus pais e irmáns
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Abstract

Text databases are growing in the last years due to the widespread use of digital libraries, document databases and mainly because of the continuous growing of the Web. Compression comes up as an ideal solution that permits to reduce both storage requirements and input/output operations. Therefore, it is useful when transmitting data through a network.

Even though compression appeared in the first half of the 20^{th} century, in the last decade, new Huffman-based compression techniques appeared. Those techniques use words as the symbols to be compressed. They do not only improve the compression ratio obtained by other well-known methods (e.g. Ziv-Lempel), but also allow to efficiently perform searches inside the compressed text avoiding the need for decompression before the search. As a result, those searches are much faster than searches inside plain text.

Following the idea of word-based compression, in this thesis, we developed four new compression techniques that make up a new family of compressors. They are based in the utilization of dense codes. Among these four techniques, the first two ones are semi-static techniques and the others are dynamic methods. They are called: End-Tagged Dense Code, (s, c)-Dense Code, Dynamic End-Tagged Dense Code, and Dynamic (s, c)-Dense Code.

Moreover, in this thesis, we have implemented a first prototype of a word-based byte-oriented dynamic Huffman compressor. This technique was developed with the aim of having a competitive technique to compare against our two dynamic methods.

Our empirical results, obtained from the systematic empirical validation of our compressors in real corpora, show that our techniques become a fundamental contribution in the area of compression. Since these techniques compress more, and more efficiently than other widely used compressors (e.g. gzip, compress, etc.), they can be applied to both Text Retrieval systems and to systems oriented to data

transmission.

It is remarkable that the research done in this thesis introduces a new family of compressors that is based on the use of dense codes. Even though we have only explored the beginning of this new family, the obtained results are so good that we hope that future works permit us to develop more compressors from this family.

Resumo

As bases de datos textuais están a medrar nos últimos anos debido á proliferación de bibliotecas dixitais, bases de datos documentais, e sobre todo polo grande crecemento continuado que a Web está a manter. A compresión xurde como a solución ideal que permite reducir espazo de armacenamento e operacións de entrada/saída, co conseguinte beneficio para a transmisión de información a través dunha rede.

Se ben a compresión nace na primeira parte do século XX, na pasada década aparecen novas técnicas de compresión baseadas en Huffman que usan as palabras como os símbolos a comprimir. Estas novas técnicas non só melloran a capacidade de compresión doutros métodos moi coñecidos (p.ex: Ziv-Lempel), senón que ademais permiten realizar buscas dentro do texto comprimido, sen necesidade de descomprimilo, dun xeito moito máis rápido que cando ditas buscas se fan sobre o texto plano.

Seguindo coa idea da compresión baseada en palabras, nesta tese desenvolvéronse catro novas técnicas de compresión que inician unha nova familia de compresores baseados na utilización de códigos densos. Destas catro técnicas, dúas son semi-estáticas e dúas son dinámicas. Os seus nomes son: End-Tagged Dense Code, (s, c)-Dense Code, Dynamic End-Tagged Dense Code e Dynamic (s, c)-Dense Code.

Ademais, nesta tese implementouse por primeira vez un compresor dinámico orientado a bytes e baseado en palabras que usa Huffman como esquema de codificación. Nós desenvolvemos este compresor para termos unha técnica competitiva e baseada en Huffman coa que comparar as nosas dúas técnicas dinámicas.

Os resultados empíricos obtidos da validación experimental sistemática dos nosos compresores contra corpus reais demostran que estes supoñen unha aportación fundamental no campo da compresión tanto para sistemas orientados a Text Retrieval como para sistemas orientados á transmisión de datos, xa que os

nosos compresores comprimen máis e máis eficientemente que moitos dos actuais compresores en uso (gzip, compress, etc.).

Hai que salientar que a investigación realizada nesta tese inicia unha nova familia de compresores baseados en códigos densos cuxas posibilidades están apenas a ser albiscadas, polo que esperamos que traballos futuros nos permitan desenvolver novos compresores desta familia.

Contents

List of Tables

List of Figures

1

Introduction

1.1 Text Compression

Compression techniques exploit redundancies in the data to represent them using less space [BCW90]. The amount of document collections has grown rapidly in the last years, mainly due to the widespread use of Digital Libraries, Document Databases, office automation systems, and the Web. These collections usually contain text, images (that are often associated with the text) and even multimedia information such as music and video.

For example, a digital newspaper treats a huge amount of news a day. Each article is composed of some text and it often encloses one or more photos, audio files, and even videos. If we consider a Digital Library that allows access to antique books, it is necessary to maintain not only digital photos of the books, which permits to appreciate the shape of the original copies, but also the text itself, which allows to perform searches about their content.

This work does not deal with the management of Document Databases in general. It is focused on natural language *Text Databases*. That is, we considered documents that contain only text. Text content is specially important if we consider the necessity of retrieving some elements from a document collection. Retrieval systems usually allow users to ask for documents containing some text (i.e. "I want documents which include the word *compression*"), but they do not usually permit to ask for other types of data.

Current Text Databases contain hundreds of gigabytes and the Web is measured in terabytes. Although the capacity of new devices to store data grows fast and the associated costs decrease, the size of text collections increases faster. Moreover, CPU speed grows much faster than that of secondary memory devices and networks, so storing data in compressed form reduces not only space, but also the I/O time and the network bandwidth needed to transmit it. Therefore, compression is more and more convenient, even at the expense of some extra CPU time. For example, if we consider the low bandwidth wireless communications in hand-held devices, compression reduces transmission time and makes transmission faster and cheaper.

A Text Database is not only a large collection of documents. It is also composed of a set of structures that guarantee efficient retrieval of the relevant documents. Among them, *inverted indexes* [BYRN99, WMB99] are the most widely used retrieval structures.

Inverted indexes store information about all the relevant terms in the Text Database, and basically associate those terms with the positions where they appear inside the Text Database. Depending on the level of granularity of the index, that position can be either an offset inside a document (*word addressing indexes or standard inverted indexes*), a document (*document addressing indexes*) or a block (*block addressing indexes*). Standard inverted indexes are usually large (they store one pointer per text word), therefore they are expensive in space requirements. Memory utilization can be reduced by using block addressing indexes. These indexes are smaller than standard indexes because they point to blocks instead of exact word positions. Of course the price to pay is the need for sequential text scanning of the pointed blocks.

Using compression along with block addressing indexes usually improves their performance. If the text is compressed with a technique that allows direct searching for words in the compressed text, then the index size is reduced because the text size is decreased, and therefore, the number of documents that can be held in a block increases. Moreover, the search inside candidate text blocks is much faster. Notice that using these two techniques together, as in [NMN+00], the index is used just as a device to filter out some blocks that do not contain the word we are looking for. This index schema was first proposed in Glimpse [MW94], a widely known system that uses a block addressing index. On the other hand, compression techniques can be also used to compress the inverted indexes themselves, as suggested in [NMN+00, SWYZ02], achieving very good results.

Summarizing, compression techniques have become attractive methods that can be used in Text Databases to save both space and transmission time. These two goals of compression techniques correspond to two distinct compression scenarios

that are described next.

1.1.1 Compression for space saving and efficient retrieval

Decreasing the space needed to store data is important. However, if the compression scheme does not allow us to search directly the compressed text, then the retrieval over such compressed documents will be less efficient due to the necessity of decompressing them before the search. Moreover, even if the search is done via an index (and especially in either block or document addressing indexes) some text scanning is needed in the search process [MW94, NMN+00]. Basically, compression techniques are well-suited for *Text Retrieval* systems iff: *i)* they achieve good compression ratio, *ii)* they maintain good search capabilities and *iii)* they permit *direct access* to the compressed text, what enables decompressing random parts of the compressed text without having to process it from the beginning.

Classic compression techniques, like the well-known algorithms of Ziv and Lempel [ZL77, ZL78] or classic Huffman [Huf52], permit to search for words directly on the compressed text [NT00, MFTS98]. Empirical results showed that searching the plain version of the texts can take half the time of decompressing that text and then searching it. However, the compressed search is twice as slow as just searching the uncompressed version of the text. Classic Huffman yields poor compression ratio (over 60%). Other techniques such as Byte-Pair Encoding [Gag94] obtain competitive search performance [TSM+01, SMT+00] but still poor compression on natural language texts (around 50%).

Classic Huffman techniques are character-based *statistical two-pass* techniques. Statistical compression techniques split the original text into symbols[1] and replace those symbols with a codeword in the compressed text. Compression is achieved by assigning shorter codewords to more frequent symbols. These techniques need a model that assigns a frequency to each original symbol, and an encoding scheme that assigns a codeword to each symbol depending on its frequency. Returning to character based Huffman, a first pass over the text to compress gathers symbols and computes their frequencies. Then a codeword is assigned to each symbol following a Huffman encoding scheme. In the second pass, those codewords are used to compress the text. The compressed text is stored along with a header where the correspondence between the source symbols and codewords is represented. This header will be needed at decompression time.

[1] If the source text is split into characters, the compression technique is said to be a character-based one. If words are considered as the base symbols we call them word-based techniques.

An excellent idea to compress natural language text is given by Moffat in [Mof89], where it is suggested that words, rather than characters, should be the source symbols to compress. A compression scheme using a semi-static word-based model and Huffman coding achieves very good compression ratio (about 25-30%). This improvement is due to the more biased word frequency distribution with respect to the character frequency distribution. Moreover, since in Information Retrieval (IR) words are the atoms of the search, these compression schemes are particularly suitable for IR.

In [MNZBY00], Moura et al. presented a compression technique called *Plain Huffman Code* a word-based byte-oriented optimal prefix[2] code. They also showed how to search for either a word or phrase into a text compressed with a word-based Huffman code without decompressing it, in such a way that the search can be up to eight times faster than searching the plain uncompressed text. One of the keys of the efficiency is that the codewords are sequences of bytes rather than bits.

Another technique, called *Tagged Huffman Code*, was presented in [MNZBY00]. It differs from Plain Huffman in that Tagged Huffman reserves a bit of each byte to signal the beginning of a codeword. Hence, only 7 bits of each byte are used for the Huffman code. Notice that the use of a Huffman code over the remaining 7 bits is mandatory, as the flag is not useful by itself to make the code a prefix code.

Direct searches [MNZBY00] over Tagged Huffman, are possible by compressing the pattern and then searching for it in the compressed text using any classical string matching algorithm. In Plain Huffman this is not possible, as the codeword could occur in the text and yet not correspond to the pattern. The problem is that the concatenation of parts of two adjacent codewords may contain the codeword of another source symbol. This cannot happen in Tagged Huffman Code because of the bit that distinguishes the first byte of each codeword. For this reason, searching with Plain Huffman requires inspecting all the bytes of the compressed text, while the fast Boyer-Moore type searching [BM77] (that is, skipping bytes) is possible over Tagged Huffman Code.

Another important advantage of using flag bits is that they make Tagged Huffman a *self-synchronizing*[3] code. As a result, Tagged Huffman permits *direct access* to the compressed text. That is, it is feasible to access a compressed text, to find the beginning of the current codeword (synchronization), and to start decompressing the text without the necessity of processing it from the beginning.

[2] A prefix code generates codewords that are never prefix of a larger codeword. This is interesting since it makes decompression simpler and faster.

[3] Given a compressed text, it is possible to easily find the beginning of a codeword by only looking for a byte with its flag bit set to 1.

The flag bit in Tagged Huffman Code has a price in terms of compression performance: the loss of compression ratio is approximately 3.5 percentage points. Although Huffman is the optimal prefix code, Tagged Huffman Code largely underutilizes the representation. Thus, there are a many bit combinations in each byte that are not used, to guarantee the code to be a prefix code.

1.1.2 Compression for file transmission

File transmission is another interesting scenario where compression techniques are very suitable. Note that when a user requests some documents from a Text Database, these documents are first located, and then they are usually downloaded through a slow network to the user's computer.

In general, transmission of compressed data is usually composed of four processes: *compression*, *transmission*, *reception*, and *decompression*. The first two are carried out by a *sender* process and the last two by a *receiver*. This is the typical situation of a downloadable zipped document available through a Web page.

There are several interesting *real-time* transmission scenarios, where compression and transmission should take place concurrently with reception and decompression. That is, the sender should be able to start the transmission of compressed data without preprocessing the whole text, and simultaneously, the receiver should start reception and decompression as the text arrives.

Real-time transmission is handled with so-called *dynamic* or *adaptive* compression techniques. Such techniques perform a single pass over the text (so they are also called *one-pass*), therefore the compression and transmission take place as the source data is read. Notice that this is not possible in *two-pass* techniques, since compression cannot start until the first pass over the whole text has been completed. Unfortunately, this restriction makes *two-pass* codes unsuitable for real-time transmission.

In the case of dynamic codes, searching capabilities are not crucial as in the case of semi-static compression methods used in IR systems.

The first interesting statistical adaptive techniques were presented by Faller and Gallager in [Fal73, Gal78]. Such techniques are based on Huffman codes. Those methods were later improved in [Knu85, Vit87]. Since they are *one-pass* techniques, the frequency of symbols and the codeword assignment is computed and updated on-the-fly during the whole transmission process, by both sender and receiver. However, those methods were character- rather than word-oriented, and

thus their compression ratios on natural language were poor (around 60%).

Currently, the most widely used adaptive compression techniques (i.e. *gzip, compress,...*) belong to the Ziv-Lempel family [BCW90]. They obtain good compression and decompression speed, however, when applied to natural language text, the compression ratios achieved by Ziv-Lempel are not that good (around 40%). Other techniques such as PPM [CW84] or arithmetic encoding [Abr63, WNC87, MNW98] obtain better compression ratios, but they are not time-efficient.

1.2 Open problems faced in this thesis

Some open problems are interesting in both text compression for efficient retrieval and dynamic compression fields. Among them we want to emphasize the two problems that were tackled in this thesis:

1. Developing compression techniques well-suited to be integrated into Text Retrieval Systems to improve their performance. Those compression techniques should join good compression ratio and good searching capabilities. We considered that developing new codes yielding compression ratios close to those of Plain Huffman while maintaining the good Tagged Huffman direct search capabilities, would be interesting.

2. Developing powerful dynamic compression techniques well-suited for its application to natural language texts. Such techniques should be well-suited for its use in real-time transmission scenarios. Good compression ratio, and efficient compression and decompression processes are additional properties that those techniques should yield. It is well-known that adaptive Huffman-based techniques obtain poor compression ratios (they are character based) and are slow. Nowadays, there exist dynamic compression techniques that obtain good compression ratios, but they are slow. There are also other time-efficient dynamic techniques, but unfortunately, they do not obtain good compression ratios. Therefore, developing an efficient adaptive compression technique for natural language texts, joining good compression ratio and good compression and decompression speed, was also a relevant problem.

1.3 Contributions of the thesis

The first task developed in this thesis was a word-based byte-oriented statistical two-pass compression technique called End-Tagged Dense Code. This code signals the last byte of each codeword instead of the first (as Tagged Huffman does). By signaling the last byte, the rest of the bits can be used in all their 128 combinations and the code is still a prefix code. Hence, 128^i codewords of length i can be built. The last byte of each codeword can use 128 possible bit combinations (i.e those values from 128 to 255) and the rest of the bytes use the remaining byte values (i.e values from 0 to 127). As a result, End-Tagged Dense Code is a "dense" code. That is, all possible combinations of bits are used for the bytes of a given codeword. Compression ratio becomes closer to the compression ratio obtained by Plain Huffman Code. This code not only retains the ability of being searchable with any string matching algorithm (i.e. algorithms following the Boyer-Moore strategy), but it is also extremely simple to build (using a sequential assignment of codewords) and permits a more compact representation of the vocabulary (there is no need to store anything except the ranked vocabulary with words ordered by frequency). Thus, the advantages over Tagged Huffman Code are (*i*) better compression ratios, (*ii*) same searching possibilities, (*iii*) simpler and faster coding, and (*iv*) simpler and smaller vocabulary representation.

However, we show that it is possible to improve End-Tagged Dense Code compression ratio even more while maintaining all its good searchability features. (s, c)-Dense Code, a generalization of End-Tagged Dense Code, improves its compression ratio by tuning two parameters, s and c, to the word frequency distribution in the corpus to be compressed. These two parameters are: the number of values (stoppers) in a byte that are used to mark the end of a codeword (s values) and the number of values (continuers) used in the remaining bytes ($256 - s = c$). As a result, (s, c)-Dense Code compresses strictly better than End-Tagged Dense Code and Tagged Huffman Code, reaching compression ratios directly comparable with Plain Huffman Code. At the same time, (s, c)-Dense Codes retain all the simplicity and direct search and direct access capabilities of End-Tagged Dense Code and Tagged Huffman Code. As addition, both End-Tagged Dense code and (s, c)-Dense Code permit to derive interesting analytical lower and upper bounds to the compression that is obtained by D-ary Huffman codes.

In the text transmission field, our goals were to introduce dynamism into word-based semi-static techniques. With this aim, three word-based dynamic techniques were developed.

We extended both End-Tagged Dense Code and (s, c)-Dense Code to build

two new adaptive techniques: Dynamic End-Tagged Dense Code and Dynamic (s, c)-Dense Code. Their loss of compression is negligible with respect to the *semi-static* version while compression speed is even better in the dynamic version of our compressors. This makes up an excellent alternative for adaptive natural language text compression.

A dynamic word-based Huffman method was also built to compare it with both Dynamic End-Tagged Dense code and Dynamic (s, c)-Dense Code. This Dynamic word-based Huffman technique is also described in detail because it turns out to be an interesting contribution. Since it is a Huffman method, it compresses slightly better than Dynamic (s, c)-Dense Code and Dynamic End-Tagged Dense Code, but it is much slower in both compression and decompression.

Specifically, the contributions of this work are:

1. The development of the End-Tagged Dense Code. It always improves the compression ratio with respect to Tagged Huffman Code, and maintains its good features: *i)* easy and fast decompression of any portion of compressed text, *ii)* direct searches in the compressed text for any kind of pattern with a Boyer-Moore approach. Empirical results comparing End-Tagged Dense Code with other well-known and powerful codes such as Tagged Huffman and Plain Huffman are also presented. End-Tagged Dense Code improves Tagged Huffman by more than 2.5 percentage points and is only 1 percentage point worse than Plain Huffman. Moreover, it is shown that End-Tagged Dense Code is faster to build than Huffman-based techniques, and it is also faster to search than Tagged Huffman and Plain Huffman.

2. The development of the (s, c)-Dense Code, a powerful generalization of End-Tagged Dense Code. It adapts better its encoding schema to the source word frequency distribution. (s, c)-Dense Code improves the compression ratio obtained by End-Tagged Dense Code by about 0.6 percentage points and maintains its good features: direct search capabilities, random access and fast decompression. We also provide empirical results comparing (s, c)-Dense Code with End-Tagged Dense Code, Tagged Huffman and Plain Huffman. With respect to Huffman-based techniques, (s, c)-Dense Code improves Tagged Huffman compression ratio by more than 3 percentage points, and its compression ratio is only 0.3 percentage points, in average, worse than the compression ratio obtained by Plain Huffman. Finally, it is also shown that (s, c)-Dense Code is faster to build and to search than the Huffman-based techniques. It is also faster in searches than End-Tagged Dense

Code, but it results slightly slower during the encoding phase. Figure 1.1 illustrates those results on an experimental setup explained in Section 2.7.

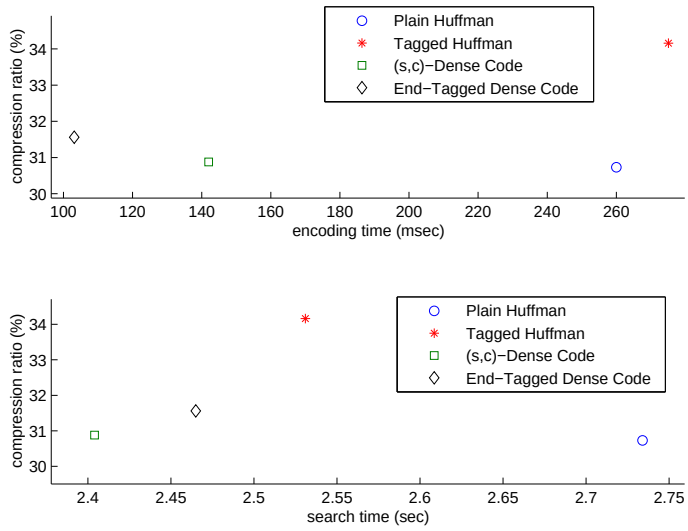


Figure 1.1: Comparison of semi-static techniques on a corpus of 564 Mbytes.

3. The derivation of new bounds for a D-ary Huffman code. Given a word frequency distribution, (s, c)-Dense Code (and End-Tagged Dense Code) can be used to provide lower and upper bounds for the *average codeword length* of a D-ary Huffman code. We obtained new bounds for Huffman techniques assuming that words in a natural language text follow a Zipf-Mandelbrot distribution.

4. The development of a word-based byte-oriented Dynamic Huffman method, which had never been implemented before (to the best of our knowledge). This is an interesting dynamic compression alternative for natural language text.

5. The adaptation of End-Tagged Dense Code to real-time transmission by developing the Dynamic End-Tagged Dense Code. It has only a 0.1% compression ratio overhead with respect to the semi-static End-Tagged Dense Code. The dynamic version is even faster at compression than the semi-static approach (around 10%), but it is much slower in decompression.

6. The adaptation of (s, c)-Dense Code to real-time transmission by developing

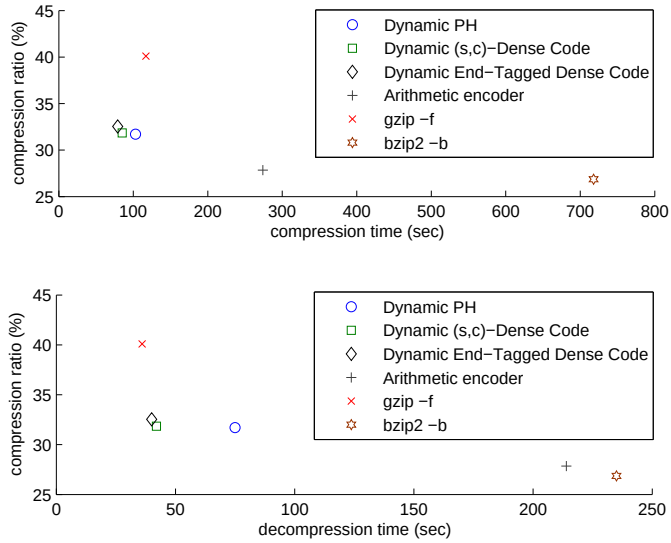


Figure 1.2: Comparison of dynamic techniques on a corpus of 564 Mbytes.

the Dynamic (s, c)-Dense Code. It has at most a 0.04% overhead in compression ratio with respect to the semi-static version of (s, c)-Dense Code. Moreover, Dynamic (s, c)-Dense Code is a bit slower (around 5%) than the Dynamic End-Tagged Dense Code, but it improves the compression ratio of the Dynamic End-Tagged Dense Code by about 0.7 percentage points.

Both Dynamic End-Tagged Dense Code and Dynamic (s, c)-Dense Code compress more than 6.5 Mbytes per second and achieve compression ratios about $30-35\%$. Comparing these results against *gzip*, our methods get an improvement in compression ratio of about $7-8$ percentage points and around 10% in compression time. However, they are slower at decompression than *gzip*. On the other hand, Dynamic (s, c)-Dense Code and End-Tagged Dense Code lose compression power with respect to *bzip2* and arithmetic coding[4], but they are much faster to build and decompress. Figure 1.2 illustrates our results.

[4]The arithmetic encoder uses a bit-oriented coding method, whereas our techniques are byte-oriented. Therefore, our methods obtain an improvement in compression and decompression speed, which implies some loss of compression ratio.

1.4 Outline

First, in Chapter 2, some basic concepts about compression, as well as a taxonomy of compression techniques, are presented. After that, following the classification of compression techniques, into well-suited to text retrieval and well-suited to transmission, the remainder of this thesis is organized in two parts.

Part one is focused on *semi-static* or *two-pass* statistical compression techniques. In Chapter 3, Compressed Text Databases, as well as the Text Retrieval systems that allow recovering documents from a Text Database, are presented. We also show how compression can be integrated into those systems. Since searches are an important part of those systems, we also introduce the pattern matching problem an describe some useful string matching algorithms.

In Chapter 4, a review of some classical text compression techniques is given. In particular, character-oriented classic Huffman code [Huf52] is reviewed. Then our discussion is focused on the word-oriented Plain Huffman and Tagged Huffman codes [MNZBY00], since these codes are the main competitors of End-Tagged Dense Code and (s, c)-Dense Code. Next, it is described how to search a text compressed with a word-oriented Huffman technique. Finally, other techniques such as Byte Pair Encoding and the Burrows-Wheeler Transform are briefly described.

In Chapter 5, End-Tagged Dense Code is fully described and tested. Chapter 6 presents the (s, c)-Dense Code. Empirical results regarding compression ratio, encoding time, and also compression and decompression time are presented. (s, c)-Dense Code is compared with both End-Tagged Dense Code and Huffman-based techniques. Finally, analytical results which yield new upper and lower bounds on the average code length of a D-ary Huffman coding are shown in Chapter 7.

Part two focuses on dynamic or one-pass compression. An introduction of classic dynamic techniques, paying special attention to dynamic Huffman codes, is addressed in Chapter 8. That chapter also includes a review of arithmetic codes, dictionary-based techniques and the predictive approach PPM.

Chapter 9 describes dynamic word-based byte-oriented Huffman code. Empirical results comparing this code and a character-based dynamic Huffman code are shown. We also compare our new dynamic Huffman-based technique with its semi-static counterpart, the Plain Huffman Code.

Chapter 10 presents the dynamic version of End-Tagged Dense Code. Its compression/decompression processes are described and compared with Huffman-

based ones.

Chapter 11 focuses on dynamic (s, c)-Dense Code. This new dynamic technique is described, and special attention is paid to show how the s and c parameters are adapted during compression. Empirical results of systematic experiments over real corpora, comparing all the presented techniques against well-known and commonly used compression methods such as *gzip*, *bzip2* and an arithmetic compressor are presented.

Finally, Chapter 12 presents the conclusions of this work. Some future lines of work are also suggested.

To complete the thesis, Appendix A enumerates the publications and research activities related to this thesis.

With the exception of our word-based dynamic Huffman code, presented in Chapter 9, that was developed to have a good dynamic Huffman code to compare with, all the remaining four codes presented form a new family of compressors. That is, End-Tagged Dense Code, (s, c)-Dense Code, and their dynamic versions, are "dense" codes. The fact of being dense gives them interesting compression properties. Hence, we consider that this new family of statistical compressors for natural language text is promising and need to be explored further as we explain in Section "Future work".

2

Basic concepts

This chapter presents the basic concepts that are needed for a better understanding of this thesis. A brief description of several concepts related to Information Theory are shown first. In Section 2.4, some well-known laws that characterize natural language text are presented: Heaps's law is shown in Section 2.4.1, and Zipf's law and Zipf-Mandelbrot's law are described in Section 2.4.2. Then a taxonomy of compression techniques is provided in Section 2.5. Some measure units that can be used to compare compression techniques are presented in Section 2.6. Finally, the experimental framework used to empirically test our compression techniques is presented in Section 2.7 and the notation used along this thesis is given in Section 2.8.

2.1 Concepts of Information Theory

Text compression techniques divide the source text into small portions that are then represented using less space [BCW90, WMB99, BYRN99]. The basic units into which the text to be compressed is partitioned are called *source symbols*. The *vocabulary* is the set of all the n distinct source symbols that appear in the text.

An *encoding scheme* or *code* defines how each source symbol is encoded. That is, how it is mapped to a *codeword*. This codeword is composed by one or more *target symbols* from a *target alphabet* Υ. The number of elements of the target alphabet is commonly $D = 2$ (binary code, $\Upsilon = \{0,1\}$). D determines the number of bits (b) that are needed to represent a symbol in Υ. If codewords are sequences of bits

(bit-oriented codewords) then $b = 1$ and $D = 2^1$. If codewords are sequences of bytes (byte-oriented codewords) then $b = 8$ and $D = 2^8$.

Compression consists of substituting each source symbol that appears in the source text by a codeword. That codeword is associated to that source symbol by the *encoding scheme*. The process of recovering the source symbol that corresponds to a given codeword is called *decoding*.

A code is a *distinct code* if each codeword is distinguishable from every other. A code is said to be *uniquely decodable* if every codeword is identifiable from a sequence of codewords. Let us consider a vocabulary of three symbols A, B, C, and let us assume that the encoding scheme maps: $A \mapsto 0$, $B \mapsto 1$, $C \mapsto 11$. Then such code is a *distinct code* since the mapping from source symbols to codewords is one to one, but it is not *uniquely decodable* because the sequence 11 can be decoded as BB or as C. For example, the mapping $A \mapsto 1$, $B \mapsto 10$, $C \mapsto 100$ is *uniquely decodable*. However, a *lookahead* is needed during decoding. A bit 1 is decoded as A if it is followed by another 1. The sequence 10 is decoded as B, if it is followed by another 1. Finally the sequence 100 is always decoded as C.

A uniquely decodable code is called a *prefix code* (or prefix-free code) if no codeword is a proper prefix of another codeword. Given a vocabulary with source symbols A, B, C, the mapping: $A \mapsto 0$, $B \mapsto 10$, $C \mapsto 110$ produces a *prefix code*.

Prefix codes are *instantaneously decodable*. That is, an encoded message can be partitioned into codewords without the need of using a lookahead. This property is important, since it enables decoding a codeword without having to inspect the following codewords in the encoded message. This improves decompression speed. For example, the encoded message 010110010010 is decoded univocally as $ABCABAB$.

A *prefix code* is said to be a *minimal prefix code* if, being x a proper prefix of some codeword, then $x\alpha$ is either a codeword or a proper prefix of a codeword, for each target symbol α in the target alphabet Υ. For example, the code that maps: $A \mapsto 0$, $B \mapsto 10$ and $C \mapsto 110$ is not a minimal prefix code because 11 is a proper prefix of 11$\underline{0}$, but 11$\underline{1}$ is neither a codeword nor a prefix of a longer codeword. If the map $C \mapsto 110$ is replaced by $C \mapsto 11$ then the code becomes a *minimal prefix code*. The minimality property avoids the use of codewords longer than needed. Figure 2.1 exemplifies the types of codes described above.

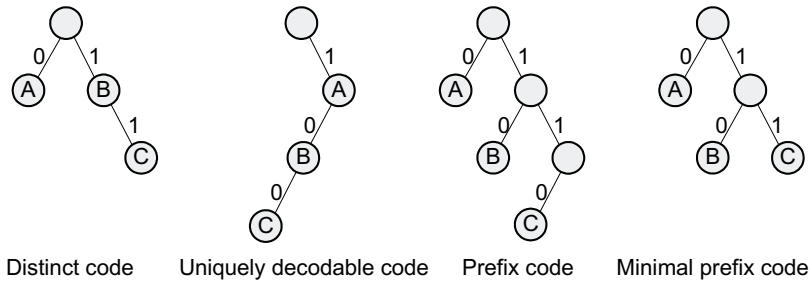


Figure 2.1: Distinct types of codes.

2.1.1 Kraft's inequality

To find a prefix code with some codeword lengths, it is important to know in which situations it is feasible to find such a code. Kraft's theorem [Kra49] presented in 1949, gives some information about that feasibility.

Theorem 2.1 *There exists a binary prefix code with codewords* $\{c_1, c_2, ..., c_n\}$ *and with corresponding codeword lengths* $\{l_1, l_2, ..., l_n\}$ *if and only if* $\sum_{i=1}^{n} 2^{-l_i} \leq 1$.

That is, if Kraft's inequality is satisfied, then a prefix code with those codeword lengths $\{l_1, l_2, ..., l_n\}$ exists. However, note that this does not imply that any code which satisfies Kraft's inequality is a prefix code.

For example, on the one hand, the *uniquely decodable code* in Figure 2.1 has codeword lengths $\{1, 2, 3\}$ and satisfies Kraft's inequality since $2^{-1} + 2^{-2} + 2^{-3} = \frac{7}{8} \leq 1$, but it is not a prefix code. On the other hand, using those codeword lengths we can build a prefix code: $A \mapsto 0$, $B \mapsto 10$, $C \mapsto 110$ as it is shown in Figure 2.1.

Moreover, it is also clear that in the case of non-prefix codes, Kraft's inequality can be unsatisfied. For example, in the *distinct code* in Figure 2.1 we have: $2^{-1} + 2^{-1} + 2^{-2} = \frac{5}{4} > 1$. Note that it is not possible to obtain a prefix code with codeword lengths $\{1, 1, 2\}$, since either the first or the second codeword would be a prefix of the third one.

Note also that when $\sum_{i=1}^{n} 2^{-l_i} = 1$, the codeword length is minimal, therefore a minimal prefix code exists.

2.2 Redundancy and compression

Compression techniques are based on reducing the redundancy in the source messages, while maintaining the source information. In [Abr63] a measure of the information content in a source symbol x_i was defined as $I(x_i) = -\log_D p(x_i)$, where D is the number of symbols of the target alphabet ($D = 2$, if a bit-oriented technique is used) and $p(x_i)$ is the probability of occurrence of a symbol x_i. This definition assumes that $p(x_i)$ does not depend on the symbols that appeared previously. From the definition of $I(x_i)$, it can be seen that:

- If $p(x_i)$ is high ($p(x_i) \to 1$) then the information content of x_i is almost *zero* since the occurrence of x_i gives very little information.

- If $p(x_i)$ is low ($p(x_i) \to 0$) then x_i is a source symbol which does not usually appear. In this situation, the occurrence of x_i has high information content.

In association with the information content of a symbol x_i, the *average information content* of the source vocabulary can be computed by weighting the information content of each source symbol x_i by its probability of occurrence $p(x_i)$. The following expression yields:

$$H = -\sum_{i=1}^{n} p(x_i) \ \log_D p(x_i)$$

Such expression is called the *entropy* of the source [SW49]. The entropy gives a lower bound to the number of target symbols that will be required to encode the whole source text.

As shown, compression techniques try to reduce the redundancy of the source messages. Having $l(x_i)$ as the length of the codeword assigned to symbol x_i, *redundancy* can be defined as follows:

$$R = \ \sum_{i=1}^{n} p(x_i)l(x_i) - H = \sum_{i=1}^{n} p(x_i)l(x_i) - \sum_{i=1}^{n} -p(x_i) \ \log_D p(x_i)$$

Therefore, redundancy is a measure of the difference between the *average codeword length* and the entropy. Since entropy takes a fixed value for a given distribution of probabilities, a good code has to reduce the average codeword length. A code is said to be a *minimum redundancy code* if it has minimum codeword length.

2.3 Entropy in context-dependent messages

Definitions in previous section treat source symbols assuming independence in their occurrences. However, it is usually possible to model the probability of the next source symbol x_i in a more precise way, by using the source symbols that have appeared before x_i.

The *context of a source symbol* x_i is defined as a fixed length sequence of source symbols that precede x_i.

Depending on the length of the *context* used, different models of the source text can be made. When that *context* is formed by m symbols, it is said that an *m-order model* is used.

In a zero-order model, the probability of a source symbol x_i is obtained from its number of occurrences. When an m-order model is used to obtain that probability, the obtained compression is better than when a lower-order model is used.

Depending on the order of the model, the *entropy* expression varies:

- *Base-order models.* In this case, it is considered that all the source symbols are independent and their frequency is uniform. Then $H_{-1} = \log_D n$.
- *Zero-order models.* In this case, all the source symbols are independent and their frequency consists of their number of occurrences. Therefore, $H_0 = -\sum_{i=1}^{n} p(x_i) \log_D p(x_i)$.
- *First-order models.* The probability of occurrence of the symbol x_j conditioned by the previous occurrence of the symbol x_i is denoted by $P_{x_j|x_i}$ and the *entropy* is computed as: $H_1 = -\sum_{i=1}^{n} p(x_i) \sum_{j=1}^{n} P_{x_j|x_i} \log_D(P_{x_j|x_i})$.
- *Second-order models.* The probability of occurrence of the symbol x_k conditioned by the previous occurrence of the sequence $x_i x_j$ is denoted by $P_{x_k|x_j,x_i}$ and the *entropy* is computed as:
  $H_2 = -\sum_{i=1}^{n} p(x_i) \sum_{j=1}^{n} P_{x_j|x_i} \sum_{k=1}^{n} P_{x_k|x_j,x_i} \log_D(P_{x_k|x_j,x_i})$.
- *Higher-order models* follow the same idea.

Several distinct m-order models can be combined to estimate the probability of the next source symbol. In this situation, it is mandatory to choose a method that describes how the probability estimation is done. In [CW84, Mof90, BCW90], a technique called *Prediction by Partial Matching* (PPM), which combines several finite-context models of order 0 to m, is described.

CORPUS	K	β		CORPUS	K	β
FT91	7.411	0.560		CALGARY	2.829	0.630
FT92	11.123	0.535		CR	13.518	0.512
FT93	11.325	0.532		ZIFF	7.471	0.546
FT94	10.507	0.535		AP	18.690	0.493
ALL_FT	8.750	0.548		ALL	1.897	0.624

Table 2.1: Parameters for Heaps' law in the experimental framework.

2.4 Characterization of natural language text

In natural language text compression, it is interesting to know how many different source symbols can appear for a given text, and also to be able to estimate the frequency of those symbols. In this section we present *Heaps' law*, which gives an approximation of how a vocabulary grows as the size of a text collection increases. We also show *Zipf's law* and *Zipf-Mandelbrot's law*. They give an estimation of the word frequency distribution for a natural language text.

2.4.1 Heaps' law

Heaps' law establishes that the relationship between the number of words in a natural language text (N) and the number of different words (n) in that text (that is, words in the vocabulary) is given by the expression $n = \alpha N^{\beta}$, where α and β are free parameters empirically determined. In English text corpora, it typically holds that $10 \leq \alpha \leq 100$ and $0.4 \leq \beta \leq 0.6$.

For natural language text corpora, Heaps's law also predicts the vocabulary size (n) from the size of the text in bytes ($tSize$), such that, $n = K \times tSize^{\beta}$. In Section 2.7, we describe ten corpora that are used in our experimental framework. Figure 2.2 illustrates the relationship between n and $tSize$ for two of those text corpora: AP Newshire 1998 (AP) and Financial Times 1994 (FT94). In corpus AP, it holds that $n = 18.690 \times tSize^{0.493}$ and in corpus FT94, $n = 10.507 \times tSize^{0.535}$. Table 2.1 shows the parameters K and β for all the corpora in the experimental framework.

The parameter β depends on the homogeneity of the corpus: the larger the β, the more heterogenous the corpus. Therefore, larger values of β imply a faster growth of the vocabulary size. For example, we have estimated from the growth of the vocabulary size in AP and FT94 corpora that, to have a vocabulary with $2,000,000$ words, AP corpus should have 700 Gbytes of text and FT94 corpus around 120 Gbytes.

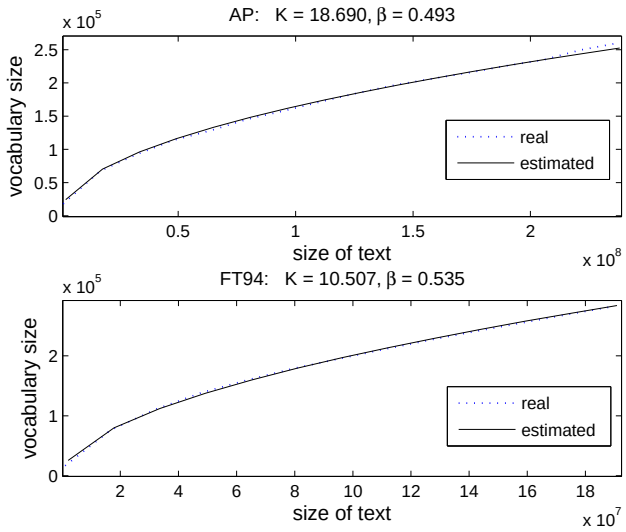


Figure 2.2: Heaps' law for AP (top) and FT94 (bottom) text corpora.

2.4.2 Zipf's law and Zipf-Mandelbrot's law

Zipf's Law [Zip49] gives a good estimation for the word frequency distribution in natural language texts. It is well-known [BCW90] that, in natural language, the probability of occurrence of words in a vocabulary closely follows Zipf's Law. That is:

$$p_i = \frac{A_z}{i^\theta}$$

Where i is the rank of a word in the vocabulary ($i = 1 \,..\, n$), θ is a constant that depends on the analyzed text ($1 < \theta < 2$), and $A_z = \frac{1}{\sum_{i>0} 1/i^\theta} = \frac{1}{\zeta(\theta)}$ is a normalization factor[1].

In [Man53] it is provided a modification of Zipf's law that is called Zipf-Mandelbrot's law. This law modifies Zipf's distribution by adding a new parameter C, which also depends on the text, in such a way that the probability of the i^{th} most frequent word in a vocabulary is given by the following expression:

$$p_i = \frac{A}{(C+i)^\theta}$$

Mandelbrot's modification fits more adequately than the original Zipf's

[1] $\zeta(x) = \sum_{i>0} 1/i^x$ is known as the Zeta function.

distribution the region corresponding to the more frequent words ($i < 100$) [Mon01]. The generalized Mandelbrot's law can be rewritten as follows:

$$p_i = \frac{A}{(1+Ci)^\theta} \tag{2.1}$$

where the parameter C needs to be adjusted to fit the data and cannot tend to *zero*.

In this case, the normalization factor A, which depends on the two parameters C and θ, is defined as:

$$A = \frac{1}{\sum_{i\geq 1} \frac{1}{(1+Ci)^\theta}} = \frac{1}{\zeta_C(\theta)} \tag{2.2}$$

Figure 2.3 shows the real probability distribution of the first 500 words in the ALL corpus (see Section 2.7). The probabilities estimated by assuming Zipf-Mandelbrot's law with $C = 0.75$ and the optimal value of the parameter θ ($\theta = 1.46$) and with other values for both θ and C are also shown. Finally, the estimation given by Zipf's law ($\theta = 1.67$) is also shown. It can be seen that Zipf-Mandelbrot's distribution gives a better estimation of the real probabilities of the source symbols than Zipf's distribution.

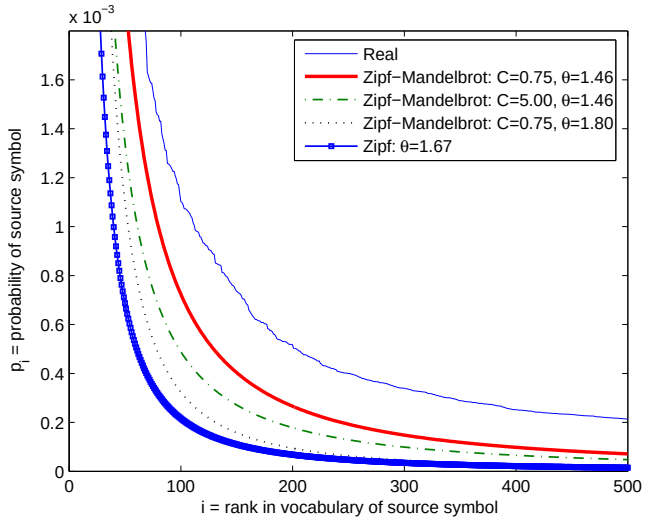


Figure 2.3: Comparison of Zipf-Mandelbrot's law against Zipf's law.

2.5 Classification of text compression techniques

Text compression is based on processing the source data to represent them in such a way that space requirements decrease [BCW90]. As a result, the source information is maintained, but its redundancy is reduced. Decompressors act over the compressed data to recover the original data.

In some scenarios (such as image or sound compression), some loss of source information can be permitted during compression because human visual/auditive sensibility cannot detect small differences between both the original and the decompressed data. In those cases, it is said that *lossy compression techniques* are used. However, if text compression is carried out, *lossless techniques* are needed. This is because it is mandatory to recover the same original text after decompression.

In order to compress a text, it is first necessary to divide the input text into symbols and to compute their probability. This process creates a representation of the text. Then an *encoding scheme* is used to assign a codeword to each symbol according to that representation. For example, both Plain Huffman and End-Tagged Dense Code use the same word-based zero-order model[2] but they use different encoding schemes (these are described in Sections 4.2 and 5.2 respectively).

The correspondence between symbols and codewords has also to be known by the decompressor, in order to be able to recover the original source data. Depending on the model used, compression techniques can be classified as using:

- *Static or non-adaptive models.* The assignment of frequencies to each source symbol is fixed. They have probability tables previously computed (usually based on experience) that are used during the encoding process. Since those probabilities are fixed, they can match badly with source data in general, so these techniques are usually suitable only in specific scenarios. Examples of this approach are the *JPEG* image standard or the *Morse* code.

- *Semi-static models.* They are usually used along with *two-pass* techniques. These methods perform a first pass over the source text in order to obtain the probabilities of all the distinct source symbols that compose the *vocabulary*. Then those probabilities remain fixed and are used during the second pass, where the source text is processed again and each source symbol is assigned a codeword whose length depends on its probability.

[2] That is, both of them divide the source text into words (word-based model), and the probability of those words is given by their number of occurrences (zero-order model).

These techniques present two main disadvantages. The first one is that the source text has to be processed twice, and therefore encoding cannot start before the whole first pass has been completed. As a result, it is impossible to apply semi-static techniques to compress text streams. The second problem relies on the necessity of providing the vocabulary and the probabilities (or the codewords), obtained by the compressor during the first pass, to the decompressor. Even though this overhead is not a problem when large texts (compared to their vocabulary) are compressed[3], it causes an important loss of compression ratio when *semi-static* techniques are applied to small texts.

Huffman-based codes [Huf52] are the main representatives of compressors that use a *semi-static* model.

- *Dynamic or adaptive models.* Compression techniques using dynamic models are commonly known as *one-pass* techniques. Instead of performing a initial pass to obtain the probabilities of the source symbols, these techniques commonly start with an initial empty vocabulary. Then the source text is read one symbol at a time. Each time a symbol is read, it is encoded using the current frequency distribution and its number of occurrences is increased. Obviously when a *new symbol* is read, it is appended to the vocabulary. The compression process *adapts* the codeword of each symbol to its frequency as compression progresses.

  In *one-pass* techniques, the decompressor adapts the mapping between symbols and codewords in the same way the compressor does. This adaptation can be done by just taking account of the sequence of symbols that were already decoded. As a consequence, it is not needed to explicitly include that mapping apart from the compressed data. This property gives *one-pass techniques* their main advantage: their ability to compress streams of text.

  Techniques such as the Ziv-Lempel family [ZL77, ZL78, Wel84], arithmetic encoding[4] [Abr63, WNC87, MNW98] and PPM [CW84] are well-known dynamic compression techniques.

Another classification of compression techniques can be done depending on how the encoding process takes place. Two families are defined: *statistical* and *dictionary* based techniques.

- *Statistical methods* assign to each source symbol a codeword whose length

[3] As Heaps' law shows (see Section 2.4.2), in a word-based model the size of the vocabulary is negligible compared to the compressed text size.

[4] Arithmetic encoding is typically used along with a dynamic model, even when static and semi-static models are also applicable.

depends on the probability of the source symbol. Compression is obtained because shorter codes are assigned to the more frequent symbols. Some typical statistical techniques are the Huffman codes and arithmetic methods.

- *Dictionary techniques* build a dictionary during the compression, storing the last appeared phrases (sequences of source symbols). Encoding is performed by substituting those phrases by small *fixed length* pointers to their position in the dictionary. Compression is achieved as the result of substituting large phrases by a *small fixed length pointer*. Compression methods from the Ziv-Lempel family are the most commonly used dictionary techniques.

2.6 Measuring the efficiency of compression techniques

In order to measure the efficiency of a compression technique, two basic concerns have to be taken into account: the performance of the algorithms involved and the compression achieved.

The complexity of compression and decompression algorithms gives an idea of how a technique will behave, but it is also necessary to obtain empirical results that permit comparing directly the performance of such technique with other methods in real scenarios. Performance is measured in time and in speed:

- *Compression and decompression times* are usually measured in seconds or milliseconds.
- *Compression and decompression speed* measure the throughput achieved. Common speed units are, Kbytes per second and Mbytes per second.

Assuming that i is the size of the input text (in bytes), that the compressed text occupies o bytes, and that b_i is the average number of bits used to represent a symbol in the source text (since the source alphabet contains typically characters, it often holds that $b_i = 8$), there are several ways to measure the compression achieved by a compression technique. The most usual measures are:

- *Compression ratio* represents the percentage that the compressed text occupies with respect to the original text size. It is computed as:

$$compression\ ratio = \frac{o}{i} \times 100$$

- *Compression rate* indicates the decrease of space needed by the compressed text with respect to the source text. It is computed as:

$$compression\ rate = \quad 100 - compression\ ratio = \frac{i-o}{i} \times 100$$

- *Bits per symbol (bps).* It compares the number of bits that are needed to represent a source symbol against the number of bits used to represent its codeword. It is computed as:

$$bps = \quad \frac{o}{i} \times b_i$$

It is interesting to point out the apparently obvious difference between absolute and relative measures when comparing two compression techniques. Next example clarifies both measurements.

Example 2.1 Let us assume two compression methods M_a and M_b that are applied to a text collection of 100 Mbytes, such that M_a compresses the text collection to 45 Mbytes and M_b to 50 Mbytes. Therefore, the compression ratios are 45% and 50% respectively. As a result, we will say that M_a improves M_b by 5 percentage points (absolute measurement). However, notice that M_a does not improve M_b by 5%. Using relative measurements, since $10 = 100 - \frac{45 \times 100}{50}$, we will say that M_a improves M_b by 10%.

2.7 Experimental framework

Several experiments were performed in this thesis in order to validate the compression achieved by the compression techniques that we developed and to compare our techniques with their competitors. All tests were carried out on an isolated dual Intel®Pentium®-III 800 Mhz system, with 768 MB SDRAM-100Mhz. It ran Debian GNU/Linux (kernel version 2.2.19). The compiler used was gcc version 3.3.3 20040429 and `-O9` compiler optimizations were set.

Time results (implied in compression, decompression and search speed) measure CPU user time. That is, those measures include only the amount of time the computer spends with the program, but the time needed to access files is excluded. Due to small variations in the running times, an average calculated from 5 runs was taken for most of the experiments. With respect to the compression effectiveness, compression ratio was measured.

Several text collections of varied sizes were chosen for this research. As a representative for short texts, we used the Calgary corpus[5]. We also used some large text collections from the Text REtrieval Conference[6] (TREC). From TREC-2, both AP Newswire 1988 and Ziff Data 1989-1990 corpora were used. From TREC-4, we used Congressional Record 1993, and Financial Times 1991, 1992, 1993, and 1994. Finally, two larger collections, ALL_FT and ALL, were used. ALL_FT includes all texts from Financial Times collection, whereas ALL collection is composed by Calgary corpus and all texts from TREC-2 and TREC-4.

Table 2.2 shows, for each corpus used, the size (in bytes), the number of words (N), and the number of different words; that is, the number of words in the vocabulary (n). The fifth column in that table shows the parameter θ from Zipf's law. Finally, the last three columns show the parameter θ from Zipf-Mandelbrot's law, assuming that C takes the values 0.75, 1.00, and 1.25 respectively.

CORPUS	**size** (bytes)	**#words**(N)	**voc.**(n)	θ_{Zipf}	$\theta_{ZM}^{0.75}$	$\theta_{ZM}^{1.00}$	$\theta_{ZM}^{1.25}$
CALGARY	2,131,045	528,611	30,995	1.270	1.343	1.302	1.272
FT91	14,749,355	3,135,383	75,681	1.450	1.390	1.352	1.323
CR	51,085,545	10,230,907	117,713	1.634	1.422	1.384	1.355
FT92	175,449,235	36,803,204	284,892	1.631	1.443	1.408	1.381
ZIFF	185,220,215	40,866,492	237,622	1.744	1.462	1.425	1.398
FT93	197,586,294	42,063,804	291,427	1.647	1.451	1.416	1.389
FT94	203,783,923	43,335,126	295,018	1.649	1.453	1.417	1.391
AP	250,714,271	53,349,620	269,141	1.852	1.458	1.422	1.395
ALL_FT	591,568,807	124,971,944	577,352	1.648	1.472	1.438	1.412
ALL	1,080,719,883	229,596,845	886,190	1.672	1.465	1.432	1.408

Table 2.2: Description of the collections used.

For all the compression methods that use a word-based model, the *spaceless* word model [MNZBY00] was used to model the separators. A separator is the text between two contiguous words, and it must be coded too. In the spaceless word model, if the separator following a word is a single white space, we just encode the word, otherwise both the word and the separator are encoded. Hence, the vocabulary is formed by all the different words and all the different separators, excluding the single white space.

We also use for comparison some well-known compressors such as:

- Gzip version 1.3.5 (2002-09-30). It is a patent free (under the terms of GNU General Public License, http://www.gnu.org) compression utility from the Ziv-Lempel 77 family.

[5]URL: http://www.data-compression.info/Corpora/CalgaryCorpus/

[6]TREC is an international conference where standard and well-known real corpora are used to test Text Retrieval Systems. More information can be found at http://trec.nist.gov/

- *Bzip2*, which is a freely available (http://sources.redhat.com/bzip2/), patent free data compressor. It uses the Burrows-Wheeler Transform along with Huffman coding. We used bzip2 version 1.0.2. (2001-12-30).
- An aritmetic compressor [CMN+99] that uses an adaptive zero-order word-based model.

2.8 Notation

We collect here most of the notation used throughout the thesis:

Symbol	Definition
n	Number of words in the vocabulary
N	Number of words in the source text
b	Number of bits needed to represent a symbol from the target alphabet. $b = 1$ for a bit-oriented code and $b = 8$ for a byte-oriented code.
D	Number of symbols in the target alphabet. $D = 2$ for a binary code and $D = 256$ for a byte-oriented code
s, c	Number of stoppers (s) and continuers (c)
p_i	Probability of the i^{th} most frequent source symbol
k_x	Codeword length when the number of stoppers is x
W_k^s	Number of words that can be encoded with up to k bytes using s as the number of stoppers
$H_D(X), H_0^D, H$	$-\sum_i p_i \log_D p_i$ (entropy implied by distribution X)
E_b	Average codeword length for a 2^b-ary End-Tagged Dense Code
$L_d(s,c)$	Average codeword length for (s,c)-Dense Code
T_b	Average codeword length for a 2^b-ary Tagged Huffman
$L_h(D)$	Average codeword length for a D-ary Plain Huffman

Part I

Semi-static compression

3

Compressed Text Databases

This chapter describes some of the structures and algorithms used to work with *Text Databases*, specially inverted indexes. We also introduce the importance of having efficient *Text Retrieval Systems* in order to recover relevant data stored inside Text Databases. The main advantages that compression techniques provide to Text Retrieval Systems, and the way those techniques can be integrated into their inverted indexes to improve performance, are also shown.

Finally, the problem of text searching is introduced, and some commonly used pattern matching algorithms such as Boyer-Moore, Horspool, and Shift-Or are presented.

3.1 Motivation

A *Document Database* can be seen as a very large collection of documents and a set of tools that permit managing it and searching it efficiently.

Libraries can be used as an analogy of a Document Database. They store many books, and a user can go there to find information related to an issue of interest. However, this user usually does not have enough time to review all books in the library in order to find that information. Therefore, it is necessary to give the user some kind of tool (e.g. a catalog) which enables him/her finding rapidly the most relevant books.

A *Digital Library*, the set of articles published by a digital newspaper, or the Web

in general, are examples of Document Databases. Since the number of documents stored is very large, the amount of space needed to store them is also enormous (and it usually increases along time). The use of compression techniques reduces the size of the documents, and consequently the amount of time needed to transfer them from secondary memory or across a network, at the expense of some CPU time needed for compression, and especially, decompression.

However, not only saving storage space is important. A Text Retrieval System working on a Text Database has to provide efficient search and retrieval of documents. *Inverted indexes* are the most commonly used retrieval structures in occidental languages. In a traditional library, a simple catalog which associates some *keywords* with each document is usually used as retrieval structure. In Text Databases, a full text model is used; that is, all the words that appear in each document are indexed (not only a few keywords). Therefore any word appearing at least once in any document can be searched and a more effective retrieval of documents will be achieved.

3.2 Inverted indexes

An *inverted index* is a data structure that permits fast retrieval of all the positions in a Text Database where a *search term* appears.

Basically, an inverted index maintains a *terms vector* or *vocabulary* in which all terms (usually words) are stored. For each of those terms t_i, a *list of occurrences* that keeps all the *positions* where t_i appears is also stored.

Note that depending on the *granularity* used, the length of the list of occurrences may vary. If the level of granularity used is coarse (e.g. the index tells the documents where each term appears) then the list of occurrences is much smaller than when the granularity is fine (the index tells the exact positions within documents for each term). Note that a term may appear lots of times in the same document, and with fine granularity, each occurrence has to be stored inside the inverted index. Therefore, the coarser the granularity, the smaller the size of the whole index. Using coarse granularity improves some searches [WMB99, BYRN99] and increases the possibilities of maintaining the whole index in memory, but if the exact occurrence positions are needed (for example for a proximity search), it will be necessary to perform a sequential scan of the pointed document or block.

For example, in Figure 3.1 the list of occurrences for the term “compression” stores 4 pointers. Each pointer references the exact location (document and offset

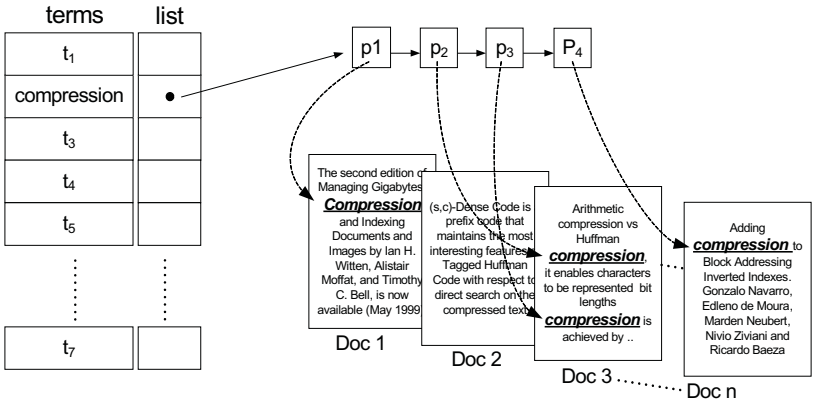


Figure 3.1: Structure of an inverted index.

inside it) of the term. If the level of granularity used for the *list of occurrences* had been the *document*, only 3 pointers would have been needed, since "*compression*" only appears inside three documents.

Searches in the inverted index start by finding in the vocabulary the search term. This initial search can be speeded up by using any suitable data structure such as hash tables or tries. This takes $O(m)$ time (being m the size of the searched pattern). Binary search is also feasible if a lexicographically ordered vocabulary is maintained. Binary search is cheaper in memory requirements and competitive in performance, since it takes $O(\log n)$ time. Finally, the list of occurrences is used to access documents.

Sometimes (e.g. with approximate searching), more than one term in the vocabulary matches the searched pattern. In this case, the initial search delivers several lists of occurrences that will be traversed in a synchronized way.

The list of occurrences is used in a different way depending on the granularity of the index:

1. word level (*word-addressing indexes*): In this case, all terms that match the searched pattern are located in the vocabulary. Then the list of occurrences indicates not only the documents, but also the offset inside the documents where those terms appear.

2. document level (*document indexes*): The list of occurrences points directly to documents. However, to find the exact position where a term appears

in a document it is necessary to sequentially search for it in all the pointed documents.

3. level of block (*block addressing indexes*) [MW94, NMN+00]. In this case the list of occurrences points to *blocks*, and a block can hold several documents. Hence, all searches require inspecting all the text in those pointed blocks in order to know in which documents the search pattern appears. Thus, there is a space-time trade-off regarding the block size.

   Block addressing indexes take special advantage of compression. Since a compressed document requires less space, more documents can be held in the same block. This reduces considerably the size of the inverted index.

When word addressing indexes are used, all searches can be performed in the index, and it is only necessary to access to the pointed documents to retrieve them. Single word searches are immediately solved since the index points to all documents where a word appears. Searching for phrase patterns (finding a sequence of words in the text) involves looking for all the words that compose the pattern, in the vocabulary of the index. Then their lists of occurrences are traversed in a synchronized way to check if the words in the phrase pattern appear together in the text. Finally the pointed documents are returned.

In the case of document addressing indexes, single word patterns can be also searched directly in the index. However, in block addressing indexes, the index points to blocks, not to documents. Therefore, it is necessary to scan the pointed blocks to know if a word appears in a given document inside the block.

It is also possible to perform phrase queries using word addressing or block addressing inverted indexes. For example, let us assume we want to find all the places where the phrase "text databases" appears, that is, the positions were "text" and "databases" appear together and "text" precedes to "databases". In this case, the index can be used to select those candidate documents/blocks where both words appear, but only a sequential search can tell whether the sequence "text databases" occurs.

As a result, in the case of document and block addressing indexes, which are those providing best space usage, keeping the text in compressed form saves not only space and transmission time for results, but also disk time for searching. In addition, direct searching the compressed text without decompressing it saves CPU time.

Compression has been used along with *inverted indexes* with good results [NMN+00, ZM95]. Text compression techniques must have some characteristics for

this symbiosis to be productive. These characteristics are explained in Section 3.3.

3.3 Compression schemes for Text Databases

Not all compression techniques are suitable for use along with Text Databases. Compression schemes have to permit two main operations: *direct access* and *direct search* in the compressed text.

- **Direct access**. The *direct access* property refers to the possibility of going to a random part of a compressed text and start decompressing from there on, avoiding to process it from the beginning. Decompression is needed for presenting results, that is, to visualize the resulting documents.

  Sometimes, we are interested in retrieving only a small context of the searched term. For example, a Web search engine presents to the user a list of relevant links and a small *snippet* of the document as the context where the searched term appears.

  If the compression scheme does not permit direct access, once a document has been located via the index, it is necessary to start decompressing the document (or a collection of texts if several documents were compressed as a whole) from the beginning.

  However, if the compression scheme permits direct access, it is possible to have a *word addressing inverted index over the compressed text*. In such case, the index returns the position inside the compressed document, and only a small portion of the compressed text, around the searched term, has to be decompressed.

  Finally, when the user chooses one link, the whole document has to be delivered to him/her and therefore the decompression of the document is mandatory. Again, if the compressed document is inside a compressed text collection, direct access avoids the necessity of starting decompression from the beginning of the compressed data.

- **Direct search**. This property is related to the feasibility of searching inside the compressed text without having to decompress it first.

  The ability to perform direct searches inside the compressed text is interesting because it permits to maintain the text in compressed form all the time, and decompression is only needed to present the retrieved documents. This saves CPU time.

For example, if a Web server answers a few requests per second, the search time is negligible with respect to the time needed to transfer a Web page from the server to a client. Therefore differences in time between performing direct searches or performing decompression first will be small. However, in the case of a Web server accepting many queries per second, the extra CPU time needed to decompress documents before the search slows down the system, causing search time to become too long.

When a compression scheme supports *direct searches*, given a pattern, it is possible to search for the compressed pattern directly into the compressed text with a (general or specialized) sequential string matching algorithm. Direct searches over text compressed with some techniques [MNZBY00] can actually improve the search performance. For example, using word-based Huffman, Moura et al. [MNZBY00] found that searching the compressed text is up to 8 times faster for approximate searches than searches on uncompressed text.

The use of a compression scheme allowing direct search permits to maintain the text in compressed form all the time, and to use *block addressing indexes* that point to compressed blocks, as it is shown in [NMN+00].

To sum up, the use of compression techniques which permit direct search and direct access turns out to be very useful. It enables maintaining the text in a compressed form all the time (which saves space), but it also improves the efficiency of Text Retrieval Systems.

There exist specialized search algorithms that enable direct searching the text compressed with techniques such as Ziv-Lempel [NT00, ABF96, FT95, NR04] and Byte Pair Encoding [Man94, STF+99].

There are also other compression techniques that enable compressing the search pattern and then performing a scan through the text to look for the compressed pattern. These compression techniques, which permit direct searching the compressed text [MNZBY00] are based on the one-to-one association between a codeword and an input symbol.

3.4 Pattern matching

The pattern matching problem consists basically in finding all the occurrences of a given pattern p in a larger text t [BM77, KMP77, Sun90]. In this section, the length of the pattern p will be denoted by m and the text length is represented as

λ. Finally, the text t is composed of symbols from an alphabet which is denoted as Σ.

Search patterns can be classified as follows:

i) Exact words. I.e. "car", "dog", etc.

ii) Prefixes and suffixes. All words that start with a specified prefix or those which end with a given suffix are searched. For example patterns "under-" and "-ation" aim to search for words such as underground, undercooked, and generation, imagination, respectively.

iii) Substrings. They obtain all words containing a sequence of characters. The pattern "-sio-" represents words such as: compression, passion, etc.

iv) Alphabetical ranges. They represent all the terms contained in a given range (assuming an alphabetical order). For example, [a, b) returns words starting with an 'a', and (zodiac, zone] returns words such as zombie and zone.

v) Patterns allowing errors[1]. For example "hot" with one error, returns words with an edit distance less or equal than 1 with respect to hot, i.e. hot, hit, hat, pot, rot, hop, shot.

vi) Regular expressions. They consist of words that can be built from an initial set of strings and three operators: union ($e_1|e_2$), concatenation (e_1e_2) and repetition zero of more times (e^*) of other regular expressions e_1, e_2, e. For example, $(red)^*(car|bike)(0|1)^*$ returns terms such as: car, car0, car1, car01, bike, redbike, ...

In this section, we are only interested in presenting the simpler string matching problem, that is, how to search for a single pattern inside a text. Three well-known and useful string matching techniques are presented. For more information about other pattern matching techniques and how to search for several patterns (*multiple string matching*) see [NR02].

The first technique shown is the Boyer-Moore [BM77] algorithm. It is briefly described here because it is the main representative of a family of fast pattern matching algorithms that is commonly used to search compressed text. In our

[1]The approximate string matching problem consists of finding all substrings in a text t that are close to the pattern p under some measure of closeness. The most common measure of closeness in Text Databases is known as the *edit distance*. A string p is said to be of *distance* k to a string q if we can transform p into q with a sequence of k insertions of single characters in arbitrary places in p, deletions of single characters in p, or substitutions of characters.

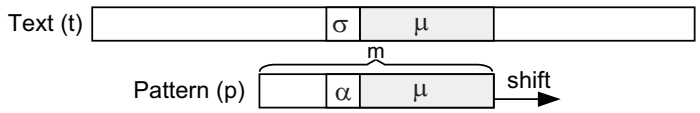


Figure 3.2: Boyer-Moore elements description.

empirical tests, we use a simpler variant of the Boyer-Moore algorithm called Horspool algorithm [Hor80]. The third technique is named Shift-Or algorithm, and it constitutes the base of the *plain filterless* algorithm which is used to search Plain Huffman code, as shown in Section 4.3.

3.4.1 Boyer-Moore algorithm

This algorithm uses a search window corresponding to the search pattern that is moved along the text. The algorithm starts by aligning the pattern p with the leftmost part of the text t, and then it compares right-to-left the pattern to the text in the window until they match (or until a difference between the pattern and the text in the window appears). In each step, the longest possible safe-shift to the right of the window is performed.

Let us assume that a maximal suffix μ of the search window is also a suffix of the pattern, and that the character σ from the text in the window does not match with character α in the pattern. Figure 3.2 shows those elements. Then three shift functions are pre-computed:

δ_1 : If the suffix μ appears in another position in p, then δ_1 is the distance from α to the next occurrence of μ which is not preceded by α backwards in the pattern. If μ does not occur again in p then $\delta_1 = m$. That is, δ_1 computes, for each suffix of the pattern, the distance to the position of its next occurrence backwards in p.

δ_2 : If the suffix μ does not appear in any text position, then suffixes ν of μ are taken into account, since they could also be a prefix of the pattern in next step. In such a case, the length of the longest prefix ν of p that is also a suffix of μ is assigned to δ_2.

δ_3 : It is associated to the distance from the rightmost occurrence of σ in the pattern to the end. If σ does not appear in p, then $\delta_3 = m$. This shift function is used because if the search window is shifted with δ_1, and σ is not aligned

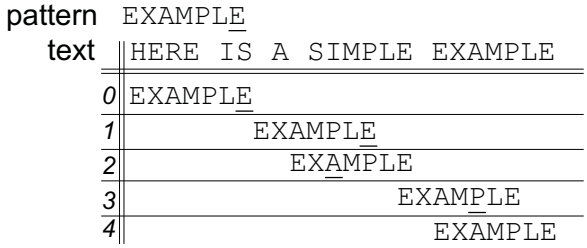


Figure 3.3: Example of Boyer-Moore searching.

with another existing σ in the pattern, then we will perform an unnecessary verification of the new search window in the next step.

Having obtained δ_1, δ_2, and δ_3 the algorithm computes two values: $M = max(\delta_1, \delta_3)$ and $min(M, m - \delta_2)$. The last value is used to slide the search window before the next iteration. However, note that when a full match of the pattern is found inside the text, only the δ_2 function is used.

An example of the way Boyer-Moore type searching works is shown in Example 3.1 and can also be followed in Figure 3.3.

Example 3.1 Let us search for the pattern `'EXAMPLE'` inside the text `'HERE IS A SIMPLE EXAMPLE'`. In step 0, the search window contains `'HERE IS'`. Since `'E'`$\neq$`'S'` then $\sigma =$`'S'`, $\alpha =$`'E'` and $\mu = \varepsilon$ (empty string). We have $\delta_1 = 1$, $\delta_2 = 0$ and $\delta_3 = m = 7$, hence the pattern is shifted 7 positions to the right.

In step 1, the search window contains `' A SIMP'`. Since `'E'`$\neq$`'P'` then $\sigma =$`'P'`, $\alpha =$`'E'` and $\mu = \varepsilon$. We have $\delta_1 = 1$, $\delta_2 = 0$ and $\delta_3 = 2$ (`'P'` appears 2 positions backwards in the pattern), hence the pattern is shifted 2 positions to the right.

In step 2, the search window contains `' SIMPLE'`. Then $\sigma =$`'I'`, $\alpha =$`'A'` and $\mu =$`'MPLE'`. We have $\delta_1 = 7$, $\delta_2 = 1$ (since the longest prefix of the pattern that is a suffix in μ is `'E'`) and $\delta_3 = 7$. Therefore the pattern is shifted $min(7, 7 - 1) = 6$ positions to the right.

In step 3, the search window contains `'E EXAMP'`. Since `'E'`$\neq$`'P'` then $\sigma =$`'P'`, $\alpha =$`'E'` and $\mu = \varepsilon$. We set $\delta_1 = 1$, $\delta_2 = 0$ and $\delta_3 = 2$ (`'P'` appears 2 positions backwards in the pattern), hence the pattern is shifted 2 positions to the right.

Finally, the pattern and the search window match in step 4. Since $\delta_2 = 1$, the pattern is shifted $7 - 1 = 6$ positions to the right and the process ends. □

The Boyer-Moore search is $O(m\lambda)$ time in the worst case. On average, its complexity is sublinear. That is, in general its performance is better when the number of elements of the alphabet ($|\Sigma|$) is larger, since the probability of matches between the symbols of the pattern and those of the search window becomes smaller, which results in large shifts of the search window. This is interesting when Boyer-Moore is applied to compressed text, where the size of the alphabet is high (usually 256 if a byte-oriented technique is used).

Variations of the initial Boyer-Moore algorithm such as the *Horspool* algorithm and the *Sunday variation* are described in [Hor80, Sun90, NR02].

3.4.2 Horspool algorithm

Horspool technique [Hor80] is a simplified version of the Boyer-Moore algorithm. It works well if the alphabet is large in comparison with the length of the search pattern. Therefore it is useful, for instance, when a byte-oriented compressed text is searched, since the alphabet has 256 symbols uniformly distributed, and the pattern usually has not more than 4 bytes.

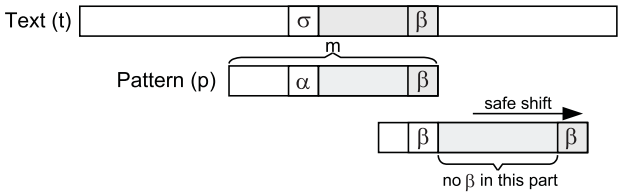


Figure 3.4: Horspool's elements description.

Horspool is distinguished from the classic Boyer-Moore algorithm by using a unique shift function. For each position in the search window, it compares its last character (β in Figure 3.4) with the last character of the pattern. If they match, the search window is verified backwards against the pattern until the whole pattern matches or we fail in a character (σ in Figure 3.4). Then whether there is a match or not, the search window is shifted according to the next occurrence of the letter β in the pattern.

Horspool algorithm consists of two parts: An initial preprocessing part and the searching phase.

1. Preprocessing phase. It computes the shift function. That is, it computes the

shift that corresponds to any symbol in the alphabet (Σ) used. If β is the final letter of the search window, the number of positions that the search window will be shifted when the backwards verification stops, is given by the distance from the last occurrence of β in the pattern p to the end of the pattern. If a symbol $\kappa \in \Sigma$ does not occur in p then the shift associated to κ is m (the length of p).

2. Searching phase. The text is traversed, and each time the pattern is aligned to the search window, a backwards verification is performed. Then the search window is shifted and the process continues.

Pseudo-code for the Horspool algorithm is given in Figure 3.5. Note that the preprocessing phase computes the shifts and stores them in a vector denoted by sv. For each symbol $\kappa \in \Sigma$, $sv[\kappa]$ keeps the shift associated to κ.

```
Preprocessing phase(p)
(1)  //input  p = p_1 p_2 p_3 ... p_m: the searched pattern
(2)  //output sv: a vector containing the shifts for each symbol
(3)  for κ ∈ Σ
(4)       sv[κ] ← m;
(5)  for i ∈ 1 ... m − 1
(6)       sv[p_i] ← m − i;
(7)  return sv;
```

```
Searching phase(p, t, sv)
(1)  //input  p = p_1 p_2 ... p_m: the searched pattern.
(2)  //input  t = t_1 t_2 ... t_λ: the text to be searched for p.
(3)  //input  sv: the vector that keeps the pre-computed shifts.
(4)  pos ← 0;
(5)  while pos ≤ λ − m
(6)       j ← m;
(7)       while j > 0 and t_{pos+j} = p_j
(8)            j ← j − 1;
(9)       if j = 0 then
(10)           occurrence appeared at position pos + 1;
(11)      pos ← pos + sv[t_{pos+m}];
```

Figure 3.5: Pseudo-code for Horspool algorithm.

An example of how Horspool algorithm works is shown in Example 3.2. It is illustrated in Figure 3.6.

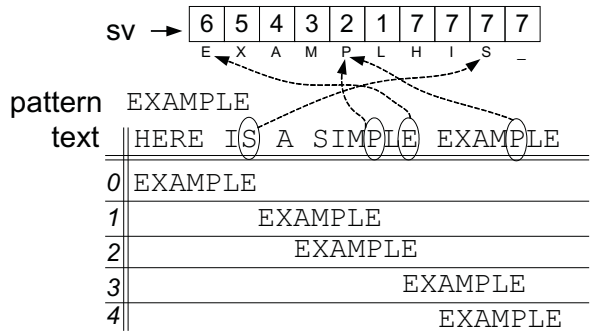


Figure 3.6: Example of Horspool searching.

Example 3.2 Let us search for the pattern `'EXAMPLE'` inside the text `'HERE IS A SIMPLE EXAMPLE'`.

Once the preprocessing phase ends, and vector sv contains the shifts that correspond to each symbol in $\Sigma = \{$`E`, `X`, `A`, `M`, `P`, `L`, `H`, `I`, `S`, `_`$\}$, the searching phase can start (see Figure 3.6).

In step 0, the search window contains `'HERE IS'`. Since `'E'`$\neq$`'S'` then $\sigma =$`'S'` and $\beta =$`'S'`, then a match cannot occur, so the search window is shifted by $sv[S] = 7$.

In step 1, the search window contains `' A SIMP'`. Since `'E'`$\neq$`'P'` then $\sigma =$`'P'` and $\beta =$`'P'`. We shift the search window $sv[P] = 2$ positions to the right.

In step 2, the search window contains `' SIMPLE'`. Then the backwards verification fails with $\sigma =$`'I'`. Since $\beta =$`'E'`, a shift of $sv[E] = 6$ positions is done.

In step 3, the search window contains `'E EXAMP'`. Since `'E'`$\neq$`'P'` then $\sigma =$`'P'` and $\beta =$`'P'`. We shift the search window by $sv[P] = 2$ positions to the right.

The pattern and the search window match in step 4. Then the pattern is shifted $sv[E] = 6$ positions to the right and the searching phase finishes. □

3.4.3 Shift-Or algorithm

The Shift-Or algorithm [WM92, BYG92] uses bitwise techniques. The algorithm acts as a nondeterministic automaton that searches for the pattern in the text. Its

key idea is to maintain a set with all the prefixes of the pattern p that match a suffix of the text read. This set is represented via a bit mask $D = d_m, d_{m-1}, \ldots, d_1$ (note that d_i represents the ith least significative bit in D).

We put a *zero* in D_j iff $p_1 \ldots p_j$ is a suffix of the text read $t_1 \ldots t_i$. If $D_m = 0$ then a match is reported. When reading the next character in the text, it is necessary to compute a new set of prefixes D'. Position $j+1$ in D' will be set to "0" iff $D_j = 0$ ($p_1 \ldots p_j$ was a suffix of $t_1 \ldots t_i$) and $t_{i+1} = p_{j+1}$. If the search pattern is not larger than the word size w of the computer (current architectures have $w = 32$ or $w = 64$ bits), then the bit mask fits completely in a CPU register. In such case, the new set D' is computed in constant time by using bitwise operations. Therefore Shift-Or algorithm is very efficient.

Being m the size of the pattern and $|\Sigma|$ the number of characters in the alphabet Σ, the algorithm starts building a table B of size $(|\Sigma| \times m)$, that keeps a bit mask $B[\kappa] = b_m \ldots b_1$ for each character κ in Σ. The bit mask in $B[\kappa]$ will have a "0" in its j^{th} position if $p_j = \kappa$. If $p_j \neq \kappa$ then $B[\kappa]_j \leftarrow 1$. That is,

$$B[\Sigma_i]_j = \begin{cases} 0 & \text{if } p_j = \Sigma_i \\ 1 & otherwise \end{cases}$$

Example 3.3 Filling the B table, when the pattern '`abbc`' is being searched inside the text '`abcabx`'.

pattern =

c	b	b	a

$B[a]$=	1	1	1	0
$B[b]$=	1	0	0	1
$B[c]$=	0	1	1	1
$B[x]$=	1	1	1	1
	4	3	2	1

Notice that, since letter '`a`' only appears in the 1st position of the pattern (starting numbering from right to left), the element $B[\mathtt{a}]_1$ is set to *zero*. The remaining elements of $B[\mathtt{a}]$ are set to 1. □

Once table B has been initialized, the search algorithm proceeds. The *mask register* D of size m is initialized with ones. A match of the current input character with the pattern, will be represented with a zero. Next, letters from the text are processed. Each time a letter κ from the text is read, D is updated as follows:

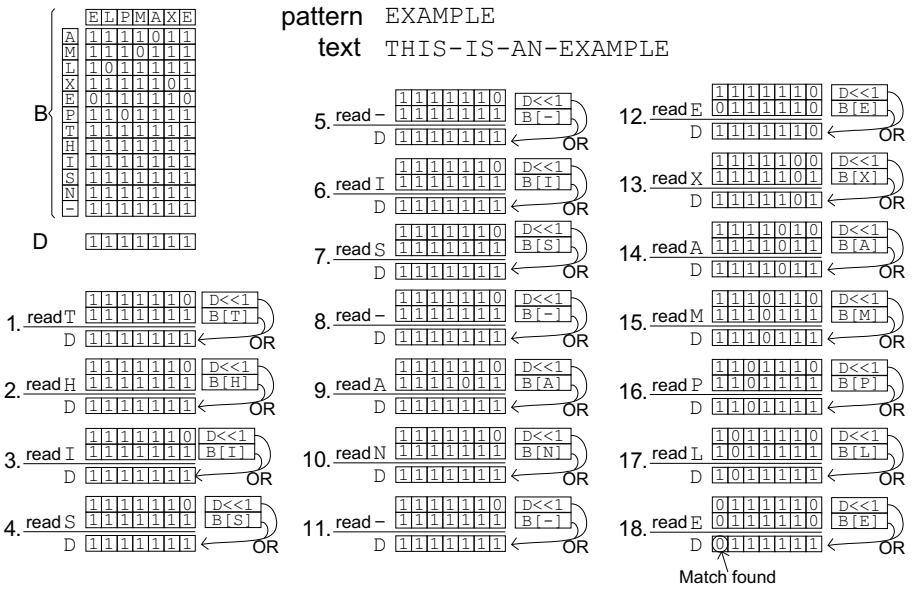


Figure 3.7: Example of Shift-Or searching.

$D \leftarrow (D << 1) \mid B[\kappa]$, where '|' indicates a bitwise OR and $B[\kappa]$ is the bit mask that corresponds to κ. A match is detected when $D[m] = 0$.

Figure 3.7 shows how Shift-Or algorithm searches for the pattern '`EXAMPLE`' inside the text '`THIS-IS-AN-EXAMPLE`'.

Assuming that the pattern length is no longer than the memory-word size of the machine ($m \leq w$), the space and time complexity of the preprocessing phase (building B table) is $O(m+z)$. The time complexity of the search phase is $O(\lambda m/w)$ in the worst case, being $O(\lambda)$ on average. Therefore, it is independent of the alphabet size and the pattern length.

3.5 Summary

In this chapter, we introduced Text Databases, and the necessity of maintaining some kind of structure to allow an efficient retrieval of documents inside a Text Database. A brief description of the *inverted index* structure was also presented.

Special attention was paid to *block addressing inverted indexes*. It was shown

how compression can be used along with this kind of inverted index to improve its performance. Some characteristics (*direct access* and *direct search* capabilities) that compression techniques must have in order to be suitable for these kind of inverted indexes were also described.

Finally, since the use of compression along with *block addressing indexes* usually requires searching the compressed text for a pattern, the pattern matching problem was presented, and three commonly used string matching techniques, the Boyer-Moore, the Horspool variant, and the Shift-Or algorithms, were also described.

4

Semi-static text compression techniques

The chapter starts by presenting the classic Huffman algorithm [Huf52], which is the base of a new word-based generation of codes that appeared in the nineties. Section 4.2, presents a brief description of some of the most interesting natural language text compression codes used nowadays. Special attention is paid to the techniques called *Plain Huffman* and *Tagged Huffman*. Section 4.3 shows how searches can be performed directly inside the text compressed with Plain Huffman or Tagged Huffman code. The chapter ends with the description of other two techniques: *Byte Pair Encoding* (BPE) and *Burrows-Wheeler Transform* (BWT). BPE is a compression technique that enables efficient search and decoding. BWT is an algorithm that transforms a text to another more compressible one.

4.1 Classic Huffman Code

Classic Huffman appeared in 1952 [Huf52]. It is a statistical semi-static technique commonly used as a character-based bit-oriented code. The codes it generates are *prefix codes*. This method produced an important break point in compression techniques. Several Huffman-based compression techniques where developed since that date [Gal78, Knu85, Vit87, MNZBY00].

Being a semi-static technique, two passes over the input text are performed: In

the first pass, the frequency of each symbol is computed. Once the frequencies of all source symbols are known, Huffman algorithm builds a tree that is used in the encoding process as explained next. This tree is the basis of the encoding scheme. In the second pass, each source symbol is encoded and output. Using the Huffman tree, shorter codes are assigned to more frequent source symbols.

The compressed file must include a header representing the vocabulary and information enough to enable the decompressor to know the shape of the Huffman tree used during compression.

4.1.1 Building a Huffman tree

The classic Huffman tree is binary: Source symbols are assigned to the leaf nodes of the tree, and their position (level) in the tree depends on their probability. Moreover, the number of occurrences of a node in a higher level can never be smaller than the number of occurrences of a node placed in a lower level.

A Huffman tree is built through the following process:

1. A set of nodes is created, one node for each distinct source symbol. Each leaf node stores a source symbol and its frequency.
2. The two least frequent nodes X and Y are taken out of the set of nodes.
3. A new internal node P is added to the set of nodes. P is set as the parent of the nodes X and Y in the tree, and its frequency is computed as the sum of the frequencies of those two nodes.
4. Steps 2) and 3) are repeated while two or more nodes remain in the set of nodes. When the whole process finishes, the unique node that remains in the set is the root of the Huffman tree (and its frequency is the sum of occurrences of all the source symbols).

Once the Huffman tree has been built, it is possible to begin the compression process. The left branch of a node is set to 0 and the right branch is set to 1. The path from the root of the tree to the leaf node where a symbol appears gives the (binary) codeword of that symbol.

Example 4.1 Building a Huffman tree from 5 input symbols: $\{a, b, c, d, e\}$ with frequencies 40, 20, 20, 15 and 5 respectively.

Figure 4.1 shows the process step by step. In the first step, a list of nodes associated to the input symbols: $\{a, b, c, d, e\}$ is created. In step two, the two least frequent nodes are chosen, and they are joined into a new internal node whose frequency is 20, that is, the sum of frequencies of d and e. In step three, the currently least frequent nodes b, c and the internal node just created could be chosen (all have the same frequency). In this case, we choose the internal node and c, and join them in a new internal node of frequency 40 that is added to the set. Note that if b had been chosen, a distinct Huffman tree would have been generated (more than one Huffman tree exist usually). Next, b and the previous internal node are joined into a new internal node, its frequency being 60. In the last step, only two nodes remained to be chosen. These two nodes are set as children of the root node. Then the branches of the Huffman tree are labelled and codewords are assigned to the symbols, as it is shown in the table in Figure 4.1. □

Assuming that the source symbols are already sorted, the cost of building a character oriented Huffman tree is $O(n)$ [MK95] where n is the number of symbols (leaf nodes) in the tree.

To decompress a compressed text, the shape of the Huffman tree used during compression has to be known. The decompression algorithm reads one bit at a time. The value of such bit permits choosing either the right or the left branch of an internal node. When a leaf is reached, a symbol has been recognized and it is output. Then the decompressor goes back to the root of the tree and restarts the process.

4.1.2 Canonical Huffman tree

In 1964, Schwartz and Kallick [SK64] defined the concept of *canonical Huffman code*. In essence, the idea is that Huffman's algorithm is only needed to compute the length of the codewords that will be mapped to each source symbol. Once those lengths are known, codewords can be assigned in several ways.

Intuitively, a canonical code builds the prefix code tree from left to right in increasing order of depth. At each level, leaves are placed in the first position available (from left to right). Figure 4.2 shows the canonical Huffman tree for Example 4.1.

The canonical code has some important properties:

- Codewords are assigned to symbols in increasing length where the lengths are

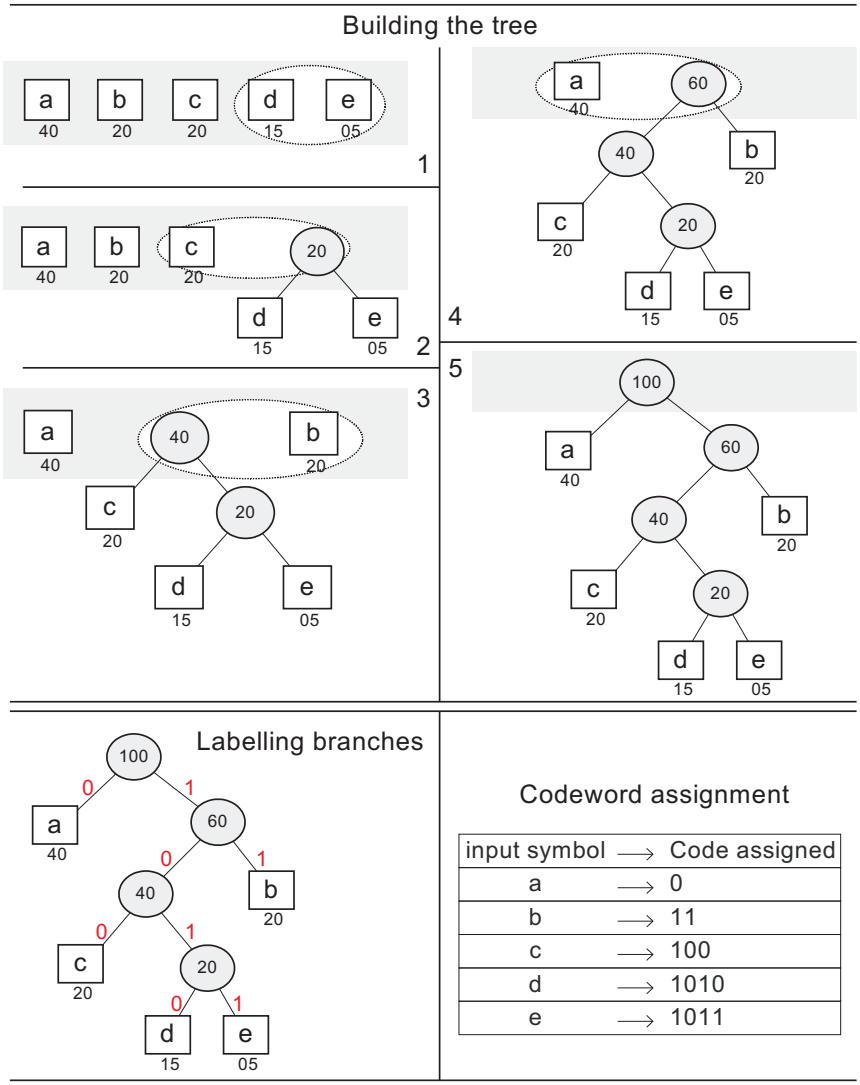


Figure 4.1: Building a classic Huffman tree.

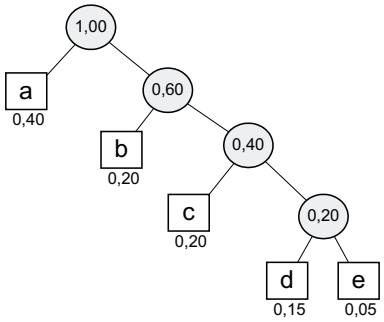


Figure 4.2: Example of canonical Huffman tree.

given by Huffman's algorithm.

- Codewords of a given length are consecutive binary numbers.

- The first codeword c_l of length l is related to the last codeword of length $l-1$ by the equation $c_l = 2(c_{l-1} + 1)$.

Given Figure 4.2, where codeword lengths are respectively 1, 2, 3, 4, and 5, the codewords obtained are: 0, 10, 110, 1110 and 1111.

The main advantage of the canonical representation is that it is possible to represent the Huffman tree by only using the lengths of the codewords. Therefore, the vocabulary needed for decompression will only require storing: *i)* the list of symbols of the vocabulary, *ii)* the *minimum* and *maximum* codeword length values and *iii)* the number of codes of each length. For example, the header of a compressed text using the Huffman tree in Figure 4.2 would be: <a, b, c, d, e><1, 4><1, 1, 1, 2>.

As a result, keeping the shape of the canonical Huffman tree of n words takes $O(h)$ integers, where h corresponds to the height of the Huffman tree. Moreover, canonical codes reduce also the memory requirements needed during compression and decompression.

Implementation details of canonical Huffman codes can be seen in [HL90]. Even when the canonical representation was defined for a bit-oriented Huffman approach, it can be also defined for a byte-oriented approach. More details about how a byte-oriented canonical Huffman code can be built appear in [MK95, MT96].

4.2 Word-Based Huffman compression

The traditional implementations of the Huffman code are character-based, that is, they use characters as the symbols of the alphabet. Therefore, compression is poor because the distribution of frequencies of characters in natural language texts is not much biased. A brilliant idea proposed by Moffat in [Mof89] is to use the words in the text as the symbols to be compressed. This idea works well because a text is more compressible when regarded as a sequence of words rather than one of characters, since in natural language text the word frequency distribution is much more biased than that of characters. Character-based Huffman methods are able to reduce English texts to approximately 60% of their original size, while word-based Huffman methods are able to reduce them to $25\% - 30\%$ of their original size.

The use of words as the basic symbols to be compressed meets the requirements of IR systems. Since words are the basic atoms in such systems, the table of symbols of an inverted index coincides with the vocabulary of the compressor, which makes it easier and faster to integrate compression with inverted indexes [WMB99, NMN+00, ZMNBY00] (the vocabulary of the compressor is needed during search and decompression).

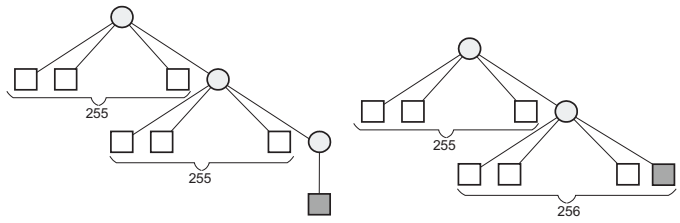
In [MNZBY00], two compression techniques that use this strategy combined with a Huffman code are presented. They are called Plain Huffman and Tagged Huffman.

As it was shown in Section 4.1, the basic method proposed by Huffman is mostly used as a binary code, that is, each symbol in the original text is coded as a sequence of bits. In [MNZBY00], they modified the code assignment such that a sequence of bytes instead of bits is associated with each word in the text. Experimental results have shown that, in natural language, there is only a small degradation in the compression ratio (less than 5 percentage points) by using bytes instead of bits. In exchange, decompression and searching are faster with byte-oriented Huffman codes because no bit manipulations are necessary.

Building a 2^b-ary Huffman tree is similar to the construction of a binary Huffman tree. The main difference stands in the start point of the process, in the number of nodes R that have to be chosen in the first iteration of the process. R was defined in [MNZBY00], as follows:

$$R = 1 + ((n - 2^b) \ mod \ (2^b - 1))$$

Then, in the following iterations of the process, 2^b nodes are taken and set as children of a new internal node, until only 2^b available nodes remain to be chosen.



(a) (b)

Figure 4.3: Shapes of non-optimal (a) and optimal (b) Huffman trees.

In general the number of iterations needed to build a 2^b-ary Huffman tree can be bounded as $\lceil \frac{n}{2^b-1} \rceil$.

Note that the expression for R given in [MNZBY00] is not optimal. If we consider a byte-oriented Huffman tree ($b = 8$) with 511 leaves, that expression gives: $R = 1 + ((511 - 256)\ mod\ (255)) = 1$. Therefore the Huffman tree would have the shape in Figure 4.3(a), and it is easy to see that such a Huffman tree is not optimal, whereas the tree shown in Figure 4.3(b) is. In order to solve this problem, we propose the following expression to compute the right R value:

$$R = \begin{cases} n & \text{if } n < 2^b \ (only\ 1\ level\ in\ the\ tree) \\ 1 + ((n - 2^b)\ mod\ (2^b - 1)) & \text{if } (n - 2^b)\ mod\ (2^b - 1) > 0 \\ 2^b & \text{if } (n - 2^b)\ mod\ (2^b - 1) = 0 \end{cases}$$

4.2.1 Plain Huffman and Tagged Huffman Codes

The two compression schemes presented in [MNZBY00] are called *Plain Huffman* and *Tagged Huffman*. Being word-oriented, both Plain Huffman and Tagged Huffman codes allow to easily search for words in the compressed text without decompressing it, in such a way that the search can be up to eight times faster for certain queries [MNZBY00]. This idea has been carried on further, up to a full integration between inverted indexes and word-based compression schemes, opening the door to a brand new family of low-overhead indexing methods for natural language texts [WMB99, NMN+00, ZMNBY00].

In [MNZBY00], they call *Plain Huffman Code* the one we have already described, that is, a word-based byte-oriented Huffman code. Plain Huffman obtains compression ratios around 30%-35% when applied to compression of English texts.

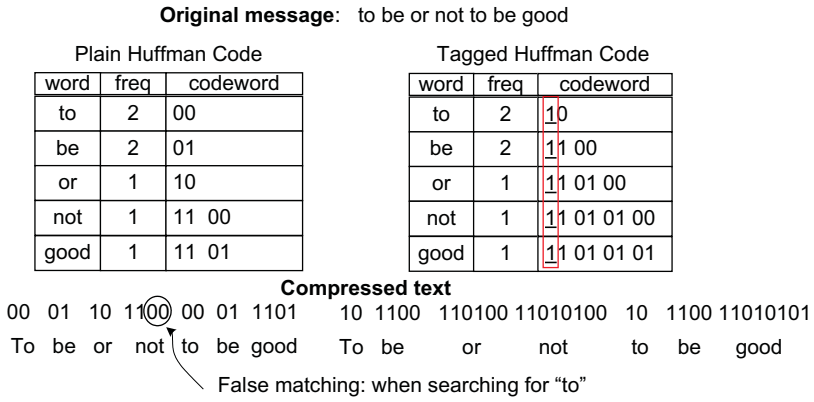


Figure 4.4: Example of false matchings in Plain Huffman.

Plain Huffman Code does not permit direct searching the compressed text by simply compressing the pattern and then using any classical string matching algorithm. This does not work because the pattern could occur in the text and yet not correspond to our codeword. The problem is that the concatenation of parts of two codewords may form the codeword of another vocabulary word. Therefore, to search for a word in a text compressed with the Plain Huffman scheme, it is necessary to read it one byte at a time. Starting in the root of the Huffman tree, the Huffman tree is traversed top-down until a leaf node is found (each byte read permits choosing a branch during the traversal). Once the algorithm gets to a leaf node, the word associated to that leaf node is the word that corresponds to the codeword just read (so it can be compared with the searched word). Then, the algorithm starts again at the root of the tree, reading the next byte in the compressed text. Searches over Plain Huffman code are explained in detail in Section 4.3.1.

The second code proposed in [MNZBY00] is called Tagged Huffman Code, which avoids that problem in searches. This technique is like the previous one, differing only in that the first bit of each byte is reserved to flag the first byte of a codeword. Hence, only 7 bits of each byte are used for the Huffman code. Note that the use of a Huffman code over the remaining 7 bits is mandatory, as the flag bit is not useful by itself to make the code a prefix code.

Therefore, due to the use of the flag bit in each byte, no spurious matchings can happen in Tagged Huffman Code. For this reason, Boyer-Moore type searching (that is, skipping bytes, Section 3.4.1) is possible over Tagged Huffman Code. See Figure 4.4 for an example of how false matchings can happen in Plain Huffman Code. Notice that, in the example, special "bytes" of two bits are used ($b = 2$).

Tagged Huffman Code has a price in terms of compression performance: full bytes are stored, but only 7 bits are used for coding. Hence, the compressed file grows approximately by 11%. As a compensation, Tagged Huffman searches compressed text much faster than Plain Huffman because Boyer-Moore type searching algorithms can be used over Tagged Huffman, as explained in Section 4.3.2.

The differences among the codes generated by the Plain Huffman Code and Tagged Huffman Code are shown in the following example.

Example 4.2 Assume that the vocabulary has 17 words, with uniform distribution ($p_i = 1/17$) in Table 4.1 and with exponential distribution ($p_i = 1/2^i$) in Table 4.2. The resulting canonical Plain Huffman and Tagged Huffman trees are shown in Figure 4.5 and Figure 4.6.

For the sake of simplicity, in this example, we consider that the used "bytes" are formed by only three bits. Hence, Tagged Huffman Code uses one bit for the flag and two for the code (this makes it look worse than it is). Flag bits are underlined in Table 4.1 and Table 4.2. □

4.3 Searching Huffman compressed text

As mentioned in the previous section, both Plain Huffman and Tagged Huffman techniques enable searching the compressed text without decompressing it. However, as shown there, no false matchings can occur in Tagged Huffman compressed text, but they can take place with Plain Huffman. Therefore searches over Tagged Huffman codes are simpler and faster that those over Plain Huffman.

4.3.1 Searching Plain Huffman Code

Two basic search methods were proposed in [MNZBY00]. The first technique is known as *plain filterless*. A preprocessing phase starts by searching for and marking in the vocabulary those words that match the search pattern (exact and complex searches are treated in the same way). In order to handle phrase patterns, a bit mask is also associated to each word. This bit mask indicates which element(s) of the pattern match(es) that word (that is, the position of the element(s) inside the pattern), and permits building a non-deterministic automaton that permits

Word	Probab.	Plain Huffman	Tagged Huffman
A	1/17	000	100 000
B	1/17	001	100 001
C	1/17	010	100 010
D	1/17	011	100 011
E	1/17	100	101 000
F	1/17	101	101 001
G	1/17	110 000	101 010
H	1/17	110 001	101 011
I	1/17	110 010	110 000
J	1/17	110 011	110 001
K	1/17	110 100	110 010
L	1/17	110 101	110 011
M	1/17	110 110	111 000
N	1/17	110 111	111 001
O	1/17	111 000	111 010
P	1/17	111 001	111 011 000
Q	1/17	111 010	111 011 001

Table 4.1: Codes for a uniform distribution.

Word	Probab.	Plain Huffman	Tagged Huffman
A	1/2	000	100
B	1/4	001	101
C	1/8	010	110
D	1/16	011	111 000
E	1/32	100	111 001
F	1/64	101	111 010
G	1/128	110	111 011 000
H	1/256	111 000	111 011 001
I	1/512	111 001	111 011 010
J	1/1024	111 010	111 011 011 000
K	1/2048	111 011	111 011 011 001
L	1/4096	111 100	111 011 011 010
M	1/8192	111 101	111 011 011 011 000
N	1/16384	111 110	111 011 011 011 001
O	1/32768	111 111 000	111 011 011 011 010
P	1/65536	111 111 001	111 011 011 011 011 000
Q	1/65536	111 111 010	111 011 011 011 011 001

Table 4.2: Codes for an exponential distribution.

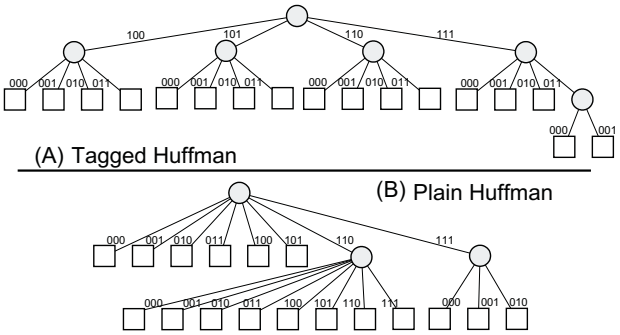


Figure 4.5: Plain and Tagged Huffman trees for a uniform distribution.

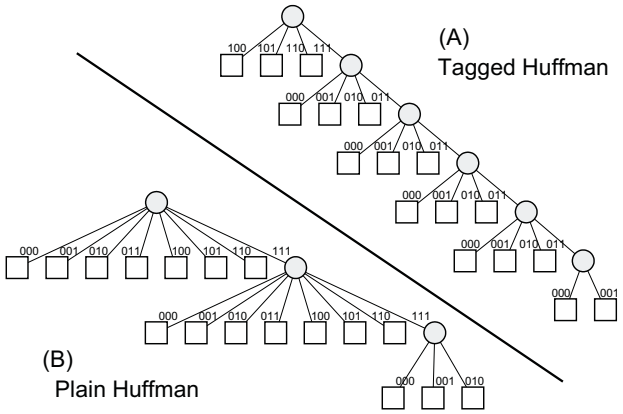


Figure 4.6: Plain and Tagged Huffman trees for an exponential distribution.

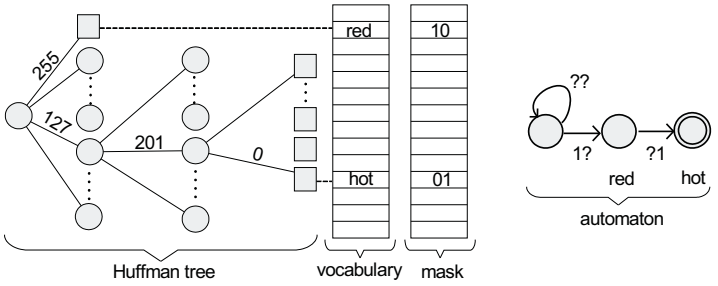


Figure 4.7: Searching Plain Huffman compressed text for pattern `"red hot"`.

recognizing the pattern. This automaton is simulated by the Shift-Or (Section 3.4.3) algorithm as shown in [BYG92].

After the preprocessing phase, the compressed text is explored one byte at a time. Each byte permits choosing a branch to traverse the Huffman tree downwards. When a leaf is reached, its bit mask is sent to the automaton, which will move from a state to another depending on the bit mask received. Figure 4.7 shows how the pattern "red hot" is searched, assuming that the codeword of "red" is [255], and that of "hot" is [127][201][0].

This is quite efficient, but not as much as the second choice, named *plain filter*, which compresses the pattern and uses any classical string matching strategy, such as Boyer-Moore. For this second, faster choice to be of use, it is necessary to ensure that no spurious occurrences are found, so the *filterless algorithm* is applied in a region where the possible match was found. To avoid processing the text from the beginning, texts are divided in small blocks during compression, in such a way that no codeword crosses a block boundary. Therefore, the filterless algorithm can start the validation of a match from the beginning of the block where that match takes place (blocks act as synchronization marks). This algorithm also supports complex patterns that are searched by using a multi-pattern matching technique. However, its efficiency may be reduced because a large number of matches, found directly by plain filter, may have to be checked by the filterless algorithm.

4.3.2 Searching Tagged Huffman Code

The algorithm to search for a pattern (a word, a phrase, etc.) under Tagged Huffman Code consists basically of two phases: compressing the pattern and then searching for it in the compressed text.

In the first phase "compressing the pattern" each element that compose the search pattern is found in the vocabulary. Then, the compressed codeword/s for the pattern is/are obtained. If a word in the pattern is not found in the vocabulary, then it cannot appear in the compressed text.

If the pattern is a word or phrase composed of single words, then the search in the vocabulary can be efficiently performed by using the appropriate data structures (i.e. a hash table). However, in the case of approximate or complex patterns (see Section 3.4), each element of the pattern (if there is more than one word) can be associated with several words in the vocabulary. To find all the words that match the pattern in the vocabulary, a sequential search is performed, and a list of codewords is held for each of the elements of the pattern.

The second phase "searching for the compressed pattern" depends also on the type of search being performed. For an exact search, the codeword of the pattern is searched in the compressed text using any classical string matching algorithm with no modifications (i.e. the Boyer-Moore algorithm presented in Section 3.4.1). This technique is called *direct search* [MNZBY00, ZMNBY00]. In the case of approximate or complex searches of one pattern, the problem of finding the codewords in the compressed text can be treated with a multi-pattern matching technique.

In the case of phrase patterns two situations are possible: The simplest case consists of searching for the single words that compose a phrase, so their codewords are obtained and concatenated. Finally this large concatenated-codeword is searched as if it were a single codeword. The second situation takes place when the elements of the phrase pattern are complex patterns, therefore more than one codeword can be associated to each element of that phrase pattern. The algorithm needs to create, for each one of the elements in the phrase pattern, a list with all its codewords. Then the elements of one of the lists are searched in the compressed text. As a heuristic, the list L_i associated to element i of the pattern is chosen so that the minimal length of the codewords in L_i is maximized [BYRN99]. Each time a match occurs, the rest of the lists are used to *validate* if that match belongs to an occurrence of the entire pattern. Note that the heuristic used is based on the idea of choosing the list whose codewords are longer, hence the number of *validations* needed will be small.

Notice that the efficiency of the previous technique gets worse when searching for frequent words, because in this situation the number of matches that occur is high, and hence many validations against the other lists have to be performed.

In [MNZBY00], they compare searching Tagged Huffman and Plain Huffman

compressed text against searching the uncompressed text. The results show that searching a compressed text for a single pattern is twice as fast as searching the uncompressed text, and that searching the compressed text for complex patterns can be up to eight times faster than searching the uncompressed text. These results have two reasons: *i)* the size of the searched data that in compressed searches is about 30% of the size of the uncompressed text, and *ii)* the vocabulary can be used to improve the searches for complex patterns. Since a major part of the work is done in the vocabulary, the scan of the text during the search is faster. Hence, compression not only saves space, but it also improves the search speed over text collections. As a result, keeping documents in compressed form and decompressing them only for presentation process seems to be very interesting.

4.4 Other techniques

In this section, two more techniques are presented. The first one is a simple compression technique, called *Byte Pair Encoding*, which offers competitive decompression speed (at the expense of bad compression ratio). The second technique is called *Burrows-Wheeler Transform*, an algorithm to transform an original text (or string) to another which is more compressible.

4.4.1 Byte Pair Encoding

Byte Pair Encoding (BPE) [Gag94], is a simple text compression technique based on pattern substitution. It is based on finding the most frequently occurring pair of adjacent letters in the text and then substituting them by a unique character that does not appear in the text. This operation is repeated until: *i)* Either all the 256 possible values of a byte are used (hence no more substitutions can be done), or *ii)* no more frequent adjacent pairs can be found (that is, all pairs appear only once).

Each time a substitution of a pair takes place, a hash table (also known as *substitution table*) is also modified either to add the new substitution pair or to increase its number of occurrences. The *substitution table* is necessary during decompression, hence the compressed file is composed of two parts: the *substitution table* and the packed data. An example of the compression process is given in Figure 4.8.

This algorithm can be considered to be multi-pass, since the number of passes over the text is not fixed, but depends on the text. The algorithm works well if

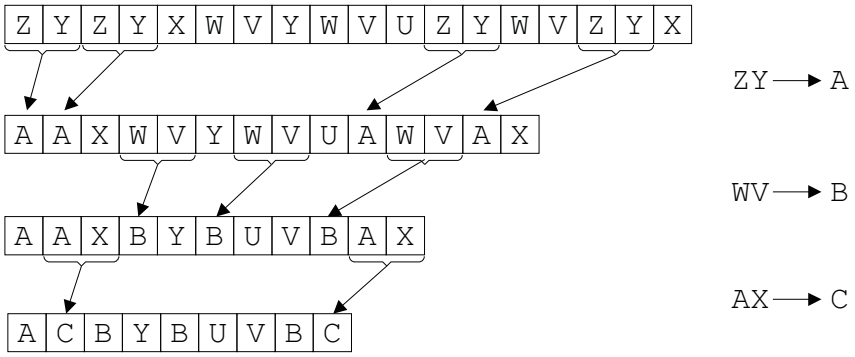


Figure 4.8: Compression process in Byte Pair Encoding.

the whole text is kept in memory. This technique could obtain poor compression when compressing large binary files where the number of unused byte values could be small, and therefore the substitution process would finish prematurely.

Those two problems (memory usage and not enough free byte values) can be avoided by partitioning the text into blocks and then compressing each block separately. This has the advantage that the *substitution table* can be locally adapted to the text in the block. However, for each compressed block, the *substitution table* has to be included along with the packed data, which worsens the compression ratio.

The three main advantages of BPE are: *i)* the high decompression speed reached (competitive with respect to *gzip*), *ii)* the possibility of partial decompression (which only needs to know the substitutions made during compression), *and iii)* the fact that it is byte-oriented. These three properties make BPE very interesting when performing compressed pattern matching. In [STF+99], two different approaches to search BPE compressed text are presented: a brute force and a Shift-Or based technique.

The main disadvantages of BPE are: *i)* very slow compression, *and ii)* bad compression ratio (comparable with character-based Huffman).

To improve compression speed, some modifications of the initial algorithm [Gag94] were proposed. In [STF+99], they achieved better compression speed than *gzip* and *compress* at the expense of a small lose in compression ratio. Basically, they compute the *substitution table* for the first block of the text, and then use it for the remainder blocks.

A word-based BPE is described in [Wan03]. This technique was proposed to efficiently permit both *phrase-based browsing* and searching of large document collections. In this scenario, two main factors are important: efficient decoding and a separate dictionary stream.

The compression algorithm is not efficient in compression time and demands large amounts of memory. Due to the high memory requirements, the source text is separated into individual blocks prior to compression. Then compression is performed on each block (as a result the compressed data and a vocabulary which contains phrases of words is obtained). Finally a post-processing *merge* stage combines the blocks in sequential phases.

They proposed two versions of word-based BPE. The first, which aims to obtain good compression ratio but does not enable *phrase-browsing*[1] is fast at decompression (comparable with *gzip*) but it is very slow at compression (about 10 times slower than *gzip*). Its main advantage is its good compression ratio (about 20-25%). The second variant is almost twice slower than the former at decompression time, and obtains worse compression ratio (about 28%). Its main advantage is the possibility of performing *phrase-browsing*.

4.4.2 Burrows-Wheeler Transform

The Burrows-Wheeler Transform (BWT) was designed by Wheeler in 1983, though it was not published until 1994 in [BW94]. It is not a compression technique, but an algorithm to transform an original text (or string) to another more compressible with certain techniques.

This algorithm is reversible. Given a string S, BWT transforms it to a new string TS such that TS contains the same data of S, but in a different order, and from TS (and little extra information) an Inverse BWT recovers the original source string S.

[1] Phrase browsing [Wan03] is a retrieval strategy that helps users to contextualize their query. This is done by first offering some phrases taken from the collection that include the answer to their query (this phrases give some context) and then allowing the user: *i)* to extend some of those phrases to obtain a larger context, or *ii)* to access the document that include a given phrase. As a result, the collection of documents can be *browsed* (by extending some contexts) and explored (by entering a given document).

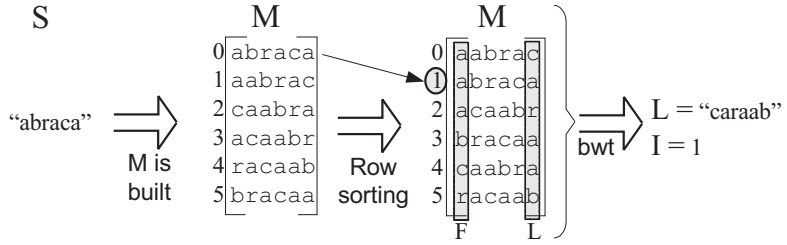


Figure 4.9: Direct Burrows-Wheeler Transform.

Computing BWT

Let S be a string of length N. The algorithm starts building a matrix $M_{N\times N}$, such that S appears in the first row, the second row contains $S >> 1$ (S circularly shifted one position to the right), and so on.

The second step consists of sorting alphabetically all the rows of the matrix, keeping track of their original position. Note that one of the rows of the sorted matrix corresponds to the initial string S. Let us call I the row containing string S.

After the sorting, the first column of M is labelled F and the last one is labelled L. Two properties hold: *i)* F contains all characters in S, but now they are sorted alphabetically, and *ii)* character j in L precedes (in S) the string contained at row j.

The result of the BWT consists of the string composed of all characters in column L of M, and the value I. That is, $BWT(S) \rightarrow (L, I)$. An example of how BWT is applied over the string 'abraca' is shown in Figure 4.9.

Computing Inverse BWT

The inverse BWT (IBWT) algorithm uses the output (L, I) of the BWT algorithm to reconstruct its input, that is, the string S of length N. IBWT consists of three phases.

In the first phase, the column F of the matrix M is rebuilt. This is done by alphabetically sorting the string L. In the example shown in Figure 4.9, $L =$ `'caraab'`. Hence, after sorting L, $F =$ `'aaabcr'` is obtained.

In the second phase, the strings F and L are used to calculate a vector T that indicates the correspondence between the characters of the two strings. That is, if $L[j]$ is the k^{th} occurrence of the character `'c'` in L, then $T[j] = i$, such that $F[i]$ is the k^{th} occurrence of `'c'` in F. Therefore vector T represents a correspondence between the elements of F and the elements of L.

Example 4.3 Note that the first occurrence of 'c'= $L[0]$ in F happens in position 4, hence $T[0] = 4$. The first occurrence of `'a'`= $L[1]$ is $F[0]$, therefore $T[1] = 0$, etc.

Position	0	1	2	3	4	5
L=	c	a	r	a	a	b
F=	a	a	a	b	c	r
T=	4	0	5	1	2	3

□

From the definition of T, it can also be seen that $F[T[j]] = L[j]$. This property is interesting because it will enable recovering the source string S.

In the last phase, the source text S is obtained using the index I, and the vectors L and T. The process that recovers the original text S starts by performing:

$$\begin{aligned} p &\leftarrow I; \\ i &\leftarrow 0; \end{aligned}$$

Then N iterations are performed in order to recover the N elements of S as follows:

$$\begin{aligned} S_{N-i-1} &\leftarrow L[p]; \\ p &\leftarrow T[p]; \\ i &\leftarrow i+1; \end{aligned}$$

Example 4.4 Having L=`'caraab'`, $T = [4, 0, 5, 1, 2, 3]$ and $I = 1$, the process starts with $p \leftarrow 1$ and $i \leftarrow 0$. Hence the first iteration makes $S_5 \leftarrow L[1] =$ `'a'`; $p \leftarrow T[1] = 0$; $i \leftarrow 1$. The second iteration makes $S_4 \leftarrow L[0] =$ `'c'`; $p \leftarrow T[0] = 4$; $i \leftarrow 2$, and so on. Notice that the string S is built right-to-left. □

Using BWT in text compression

To see why applying BWT leads to a more effective compression, let us consider the example of the letter `'t'` in the word `'the'`, and assume a source English text S containing many instances of `'the'`. When the rows in M are sorted, all those rows starting with `'he '` will appear together. A large proportion of them are likely to end in (i.e. be preceded by) `'t'`. Hence, one region of the text L will contain a very large number of `'t'` characters, along with other characters that can precede `'he'` in English, such as space, `'s'`, `'T'`, and `'S'`. The same argument can be applied to all characters in all words, so any localized region of the string L will contain a large number of a few distinct characters.

The overall effect is that the probability that a character `'t'` will occur at a given point in L is very high if `'t'` occurs near that point in L, and it is low otherwise. This property is exactly the one needed for effective compression by a *move-to-front* (MTF) encoder [BSTW86], which encodes an instance of character `'t'` by the count of distinct characters seen since the last previous occurrence of `'t'`. Basically, a MTF encoder keeps a list Z with all characters in the alphabet used. Each time a character c is processed, c is searched in Z, and the position j such as $Z_j = c$ is output. Then Z_j is moved to the front of Z (by shifting the characters Z_i, $i = 0..j-1$ to the positions $1..j$ and setting $Z_0 = c$). Therefore, if the next character in L is again the same character c, it will be replaced by a 0 in R (where R is a list of numbers generated by the MTF encoder).

As a result, consecutive repetitions of a character will become consecutive zeros in the vector R, and consecutive repetitions of a small set of characters will produce an output dominated by low numbers. For example, for some sequences taken from Calgary corpus the percentage of zeros in the sequence R may reach 90%. In [BWC89] they showed that the sequence of numbers in R will contain 60% zeros on average.

To sum up, once BWT has been applied over a source text S, and the transformed string L and the I value have been obtained, it is possible to apply a MTF encoder to transform L into a sequence of numbers R. The sequence R will be much more compressible than the initial text S. R is usually compressed using either a Huffman-based or an arithmetic encoder. However, since *zero* is the dominant symbol in R, there are many long runs in the sequence R consisting of zeros, called *zero-runs*. This led Wheeler to propose another transform called *Zero-Run Transform*, which is also known as RLE-0, to treat *0-runs* in a special way. It was not published by Wheeler but reported by Fenwick in [Fen96]. Experimental results [BK00] indicate that the application of the *RLE-0 transform* indeed improves

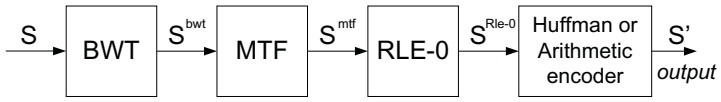


Figure 4.10: Whole compression process using BWT, MTF, and RLE-0.

the compression ratio, obtaining compression ratios around 20-25%.

Figure 4.10 summarizes the whole compression process of a text S when BWT, MTF and RLE-0 transforms are applied.

For decompression, the process starts by decompressing the compressed data (using the corresponding technique applied in compression) and then applying the IBWT process to obtain the plain text S.

Compressed indexes based on BWT

In [FM00, FM01] Ferragina and Manzini presented the *FM-index*, a compression technique that produces a compressed file which can be used as a text retrieval index. It is a sort of self-indexing[2] tool that carefully combines the Burrows-Wheeler algorithm and the suffix array[3] data structure [MM93]. The base of FM-index is the existence of an implicit suffix array in the sorted rows of matrix M that is used by BWT. Ferragina and Manzini increased the information kept by BWT in order to support efficient random access to the compressed data without having to uncompress all of it at search time.

Basically two search operations are allowed by the FM-index:

- *count* operation returns the number of occurrences of a pattern p in a text t by taking advantage of two properties of matrix M: *a)* all the suffixes of the text t prefixed by a pattern p appear in consecutive rows in M, and *b)* the set of those rows starts in position $iniP$ and ends in $endP$, such that $iniP$ is the lexicographic position of the string p among all the sorted rows of M and $endP$ indicates the last row that is prefixed by p.

- *locate* obtains, given a row index i in M, the starting position in the text t of the suffix that corresponds to the row $M[i]$.

[2]The structure of the compressed file makes it easier to search for a pattern.

[3]A suffix array is a data structure that maintains lexicographically sorted all the suffixes that appear in a text.

The FM-index achieves compression ratios close to those of the best compressors (e.g. *bzip2*) and permits fast counting of the pattern occurrences, while the cost of its retrieval is reasonable when the number of occurrences is low. More details can be found in [FM00, FM01].

BWT in a word-based approach

As it was shown, Burrows-Wheeler Transform is typically applied to obtain improved character-based compression (e.g. *bzip2*). A word-based approach has been considered in [MI04]. In that paper, Moffat and Isal proposed a compression mechanism composed of four transformations: *i)* parsing the input text into a sequence of spaceless words, representing them by integer indexes into a dictionary of strings; *ii)* applying the BWT to the sequence of integers obtained; *iii)* applying a recency-rank[4] transformation to the BWT output to obtain a more compressible sequence of integers; *and iv)* applying a Huffman-based or an arithmetic encoder to that output sequence. The resulting code obtained good compression (compression ratio about 20%) at the expense of compression and decompression speed (more than twice slower than *bzip2*, which is already slow).

4.5 Summary

In this chapter, an introduction to the classical Huffman technique as well as a description of the mechanism to build a Huffman tree, and its representation via a canonical tree, were shown. A special emphasis was put in the description of the two word-based Huffman techniques that appeared in [MNZBY00], since those techniques use the same word-based statistical approach of the semi-static codes we develop in this thesis (Chapters 5 and 6). In Section 4.3, we describe how to search directly into a text compressed with Tagged Huffman and Plain Huffman.

Finally, two further techniques were presented: Byte Pair Encoding, a compression scheme based on pattern substitution, and the Burrows-Wheeler Transform, a technique to transform a text into a more compressible one.

[4]The recency-rank operations are carried out in an elegant manner using a *splay tree*[ST85]. The simple MTF algorithm is not suitable if words, instead of characters, are considered. In a character enviroment, a linear search on the MTF list is both simple to implement and economical in operation. However, in a word-based BWT, the alphabet of source symbols is very large and locality is less pronounced, so sequential search is not feasible.

5

End-Tagged Dense Code

This chapter explains in detail the End-Tagged Dense Code. This is the first original compression technique developed in collaboration between the Database Laboratory of the University of A Coruña and the Department of Computer Science of the University of Chile. Some preliminary ideas about this compression technique were presented in [BINP03]. In this thesis, the whole procedure was developed from these initial ideas, and efficient implementations of the compressor, decompressor, and search algorithms were made. We also carried out an analytical study (presented in Chapter 7) where it is shown how to use End-Tagged Dense Code to bound Huffman compression ratio assuming that words in the natural language text follow Zipf-Mandelbrot's Law [Man53].

In Section 5.1, the motivation of the End-Tagged Dense Code is presented. Next, the way End-Tagged Dense Code works is described in Section 5.2. Later, encoding and decoding mechanisms are explained and pseudo-code is given in Section 5.3. Section 5.4 is focused on searches, explaining how to use Boyer-Moore type searching over End-Tagged Dense Code. In Section 5.5, some empirical results comparing End-Tagged Dense Code against Plain Huffman and Tagged Huffman are given. Finally, some conclusions about End-Tagged Dense Code are shown in Section 5.6.

5.1 Motivation

As it has been already pointed out in Section 4.2, Tagged Huffman has two good properties that Plain Huffman does not have: direct search capabilities and direct

#Bytes	Tagged Huffman Code	End-Tagged Dense Code
1	**1**xxxxxxx	**1**xxxxxxx
2	**1**xxxxxxx **0**xxxxxxx	**0**xxxxxxx **1**xxxxxxx
3	**1**xxxxxxx **0**xxxxxxx **0**xxxxxxx	**0**xxxxxxx **0**xxxxxxx **1**xxxxxxx
...	...	...
n	**1**xxxxxxx **0**xxxxxxx ... **0**xxxxxxx	**0**xxxxxxx ... **0**xxxxxxx **1**xxxxxxx

Table 5.1: Codeword format in Tagged Huffman and End-Tagged Dense Code.

access. Being a prefix code, Tagged Huffman does not make use of all the possible combinations of bits in each byte. Moreover, the use of a flag bit implies that its compression ratio is not optimal.

End-Tagged Dense Code uses, as Plain and Tagged Huffman do, a semi-static statistical word-based model, but it is not based on Huffman at all. Its interest lies in that it improves Tagged Huffman compression ratio and keeps all the good features of Tagged Huffman code:

- It is a *prefix code*[1].
- As Tagged Huffman code, it enables fast decompression of arbitrary portions of the text by using a flag bit in all the bytes that compose a codeword.
- It permits to use Boyer-Moore type searching algorithms (see Section 3.4.1) directly on the compressed text.

Besides, encoding and decoding with End-Tagged Dense Code are simpler and faster than with Plain Huffman and Tagged Huffman.

5.2 End-Tagged Dense Code

End-Tagged Dense Code starts with a seemingly dull change of Tagged Huffman Code. Instead of using the flag bit to signal the *beginning* of a codeword, the flag bit is used to signal the *end* of a codeword. That is, the flag bit is *0* for the first bit of any byte of a codeword except for the last one, which has a *1* in its more significative bit. This difference can be observed in Table 5.1.

[1]Even though it is a *prefix code*, End-Tagged Dense Code (as Plain Huffman) is not a *suffix code* as Tagged Huffman is. Therefore, even though it permits Boyer-Moore type searching algorithms, when a match is found, a check over the previous byte is needed. A full explanation is presented in Section 5.4.

This change has surprising consequences. Now the flag bit is enough to ensure that the code is a prefix code regardless of the contents of the other 7 bits of each byte. To see this, consider two codewords X and Y, being X shorter than Y ($|X| < |Y|$). X cannot be a prefix of Y because the last byte of X has its flag bit in 1, while the $|X|$-th byte of Y has its flag bit in 0. This fact can be easily seen in Table 5.1.

At this point, there is no need at all to use Huffman coding over the remaining 7 bits to get a *prefix code*. Therefore it is possible to use *all* the possible combinations of 7 bits in all the bytes, as long as the flag bit is used to mark the last byte of the codeword.

The encoding process is simpler and faster than Huffman, since no tree has to be built. Notice that we are not restricted to use symbols of 8 bits to form the codewords. It is possible to use symbols of b bits. Therefore, End-Tagged Dense Code is defined as follows:

Definition 5.1 *Given source symbols with decreasing probabilities $\{p_i\}_{0 \le i < n}$ the corresponding codeword using the End-Tagged Dense Code is formed by a sequence of symbols of b bits, all of them representing digits in base 2^{b-1} (that is, from 0 to $2^{b-1}-1$), except the last one which has a value between 2^{b-1} and $2^b - 1$, and the assignment is done in a sequential fashion.*

Specifically, being k the number of bytes in each codeword ($k \ge 1$) then:

$$2^{b-1}\frac{2^{(b-1)(k-1)}-1}{2^{b-1}-1} \le \ i \ < 2^{b-1}\frac{2^{(b-1)k}-1}{2^{b-1}-1}$$

The codeword corresponding to symbol i is obtained as the number x written in base 2^{b-1}, where $x = i - \frac{2^{(b-1)k}-2^{b-1}}{2^{b-1}-1}$, and adding 2^{b-1} to the last digit.

That is, using symbols of 8 bits, the encoding process can be described as follows:

- Words in the vocabulary are decreasingly ranked by number of occurrences.
- Codewords 128 to 255 (**1**0000000 to **1**1111111) are given to the first 128 words in the vocabulary. That is, to words ranked from 0 to $2^7 - 1$.
- Words ranked from $2^7 = 128$ to $2^7 2^7 + 2^7 - 1 = 16511$ are assigned sequentially to two-byte codewords. The first byte of each codeword has a value in the range $[0, 127]$ and the second in range $[128, 255]$.
- Word 16512 is assigned a tree-byte codeword, and so on.

Word rank	codeword assigned	# Bytes	# words
0	**1**0000000	1	
1	**1**0000001	1	
2	**1**0000010	1	2^7
...	...	...	
$2^7 - 1 = 127$	**1**1111111	1	
$2^7 = 128$	**0**0000000:**1**0000000	2	
129	**0**0000000:**1**0000001	2	
130	**0**0000000:**1**0000010	2	
...	...	...	
255	**0**0000000:**1**1111111	2	
256	**0**0000001:**1**0000000	2	$2^7 2^7$
257	**0**0000001:**1**0000001	2	
258	**0**0000001:**1**0000010	2	
...	...	...	
$2^7 2^7 + 2^7 - 1 = 16511$	**0**1111111:**1**1111111	2	
$2^7 2^7 + 2^7 = 16512$	**0**0000000:**0**0000000:**1**0000000	3	
16513	**0**0000000:**0**0000000:**1**0000001	3	
16514	**0**0000000:**0**0000000:**1**0000010	3	$(2^7)^3$
...	...	...	
$(2^7)^3 + (2^7)^2 + 2^7 - 1$	**0**1111111:**0**1111111:**1**1111111	3	
...	...	...	

Table 5.2: Code assignment in End-Tagged Dense Code.

As it can be seen in Table 5.2, the computation of codes is extremely simple: It is only necessary to order the vocabulary by word frequency and then sequentially assign the codewords. Hence the coding phase will be faster than using Huffman because obtaining the codes is simpler.

What is perhaps less obvious is that *the code depends on the rank of the words, not on their actual frequency.* That is, if we have four words *A, B, C, D* (ranked $i \ldots i+3$) with frequencies 0.36, 0.22, 0.22, and 0.20, respectively, then the code will be the same as if their frequencies were 0.9, 0.09, 0.009, and 0.001.

In Example 5.1, the differences among the codes generated by Plain Huffman Code, Tagged Huffman Code and End-Tagged Dense Code, are shown. For the sake of simplicity we consider that the "bytes" used are formed by only three bits ($b = 3$).

Example 5.1 A vocabulary of 17 words is used. Table 5.3 shows the codeword assigned to each word assuming that the frequencies of those words in the text follow a uniform distribution ($p_i = 1/17$). In Table 5.4 an exponential distribution ($p_i = 1/2^i$) is assumed.

Assuming "bytes" of three bits, Tagged Huffman and End-Tagged Dense Code use one bit for the flag and two for the code (this makes them look worse than they are). Flag bits are underlined. Plain Huffman uses all the three bits to the code.

Notice that in the case of End-Tagged Dense Code, the code assignment is the same independently of the distribution of frequencies of the words in the vocabulary. □

5.3 Encoding and decoding algorithms

5.3.1 Encoding algorithm

The encoding process is rather simple, as it was shown in the previous section. There are two available encoding algorithms: sequential encoding and on-the-fly encoding.

The sequential encoding algorithm computes the codewords for all the words in the sorted vocabulary and stores them in a vector that we call *codeBook*. Its pseudo-code is presented next.

```
SequentialEncode (codeBook, n)
(1)  //input n: number of codewords that will be generated
(2)  firstKBytes ← 0;    //rank of the first word encoded with k bytes
(3)  numKBytes ← 128;    //number of k-byte codewords
(4)  p ← 0;    //current codeword being generated
(5)  k ← 1;    //size of the current codeword
(6)  while p < n    //n codewords are generated
(7)      paux ← p − firstKBytes;    //relative position inside k-byte codewords
(8)      while (p < n) and (paux < numKBytes) //k-byte codewords are computed
(9)          codebook[p].code[k − 1] ← (paux mod 128) + 128; //leftmost byte
(10)         paux ← paux div 128;
(11)         for i ← k − 2 downto 0
(12)             codebook[p].code[i] ← paux mod 128;
(13)             paux ← paux div 128;
(14)         p ← p + 1;
(15)         paux ← p − firstKBytes;
(16)     k ← k + 1;
(17)     firstKBytes ← firstKBytes + numKBytes;
(18)     numKBytes ← numKBytes × 128;
```

Notice that, the operations: *mod* 128, *div* 128, + 128 and × 128 that

Word	Probab.	Plain Huffman	Tagged Huffman	End-Tagged Dense Code
A	1/17	000	100 000	100
B	1/17	001	100 001	101
C	1/17	010	100 010	110
D	1/17	011	100 011	111
E	1/17	100	101 000	000 100
F	1/17	101	101 001	000 101
G	1/17	110 000	101 010	000 110
H	1/17	110 001	101 011	000 111
I	1/17	110 010	110 000	001 100
J	1/17	110 011	110 001	001 101
K	1/17	110 100	110 010	001 110
L	1/17	110 101	110 011	001 111
M	1/17	110 110	111 000	010 100
N	1/17	110 111	111 001	010 101
O	1/17	111 000	111 010	010 110
P	1/17	111 001	111 011 000	010 111
Q	1/17	111 010	111 011 001	011 100

Table 5.3: Codes for a uniform distribution.

Word	Probab.	Plain Huffman	Tagged Huffman	End-Tagged Dense Code
A	1/2	000	100	100
B	1/4	001	101	101
C	1/8	010	110	110
D	1/16	011	111 000	111
E	1/32	100	111 001	000 100
F	1/64	101	111 010	000 101
G	1/128	110	111 011 000	000 110
H	1/256	111 000	111 011 001	000 111
I	1/512	111 001	111 011 010	001 100
J	1/1024	111 010	111 011 011 000	001 101
K	1/2048	111 011	111 011 011 001	001 110
L	1/4096	111 100	111 011 011 010	001 111
M	1/8192	111 101	111 011 011 011 000	010 100
N	1/16384	111 110	111 011 011 011 001	010 101
O	1/32768	111 111 000	111 011 011 011 010	010 110
P	1/65536	111 111 001	111 011 011 011 011 000	010 111
Q	1/65536	111 111 010	111 011 011 011 011 001	011 100

Table 5.4: Codes for an exponential distribution.

were included above to increase readability, can be made through faster bitwise operations using the following translation table:

operation		bitwise operation
$x \; mod \; 128$	$\longrightarrow$	$x \; and_b \; 0111111$
$x \; + \; 128$	$\longrightarrow$	$x \; \; or_b \; 1000000$
$x \; div \; 128$	$\longrightarrow$	$x \; >> \; 7$
$x \; \times \; 128$	$\longrightarrow$	$x \; << \; 7$

Storing the codewords in an structure such as a codebook can be avoided. Given a word rank i, we can obtain on-the-fly its ℓ-byte codeword in $O(\ell) = O(\log i)$ time. Considering that the codes are assigned in a sequential way, once words have been sorted by frequency in the vocabulary, an on-the-fly encoding algorithm can also be used. Next, the pseudo-code for the on-the-fly encoding algorithm is shown. The algorithm outputs the bytes of each codeword one at a time from right to left. That is, it begins outputting the least significative bytes first.

```
Encode (i)
(1)  //input i: rank of the word being encoded 0 ≤ i ≤ n − 1
(2)  output ((i mod 128) + 128); //rightmost byte
(3)  i ← i div 128;
(4)  while i > 0 // remainder bytes
(5)       i ← i − 1;
(6)       output (i mod 128);
(7)       i ← i div 128;
```

However, in practice, it is faster to sequentially compute the codewords for the whole words in the vocabulary (in $O(n)$ time) in such a way that the assignment word-codeword is pre-computed before the second pass of the compression starts (immediately after words in the vocabulary are ranked by frequency). As in any other two-pass statistical compression technique, after compressing the text, it is necessary to store the vocabulary along with the compressed text in order to be able to decompress it later.

Notice that there is no need to store neither the codewords (in any form such as a tree) nor the frequencies in the compressed file. It is enough to store the plain words sorted by frequency. Therefore, the vocabulary will be slightly smaller than in the case of the Huffman code, where some information about the shape of the tree must be stored (even when a canonical Huffman tree is used).

5.3.2 Decoding algorithm

The first step to decompress a compressed text is to load the words that compose the vocabulary in a vector. Since these words were stored sorted by frequency along with the compressed text during compression, the vocabulary is recovered already sorted by frequency. Once the sorted vocabulary is obtained the decoding of codewords can begin.

In order to obtain the word w_i that corresponds to a given codeword c_i, the decoder can run a simple computation to obtain the rank of the word i from the codeword c_i. Then, using the value i, the corresponding word can be obtained from $vocabulary[i]$. A code c_i of ℓ bytes can be decoded in $O(\ell) = O(\log i)$ time.

The algorithm inputs a codeword x, and iterates over each byte of x. The last byte of the codeword has the tag byte to 1, so its value is greater than 127. This permits to distinguish the end of the codeword. After the iteration, a position i is returned, and the decoded word is obtained from $vocabulary[i]$. For example, decoding a four bytes codeword $x = x_1x_2x_3x_4$ to obtain the rank i of the corresponding word is basically done as follows:

$$i \quad = \quad (((x_1 \times 128) + x_2) \times 128) + x_3) \times 128) + (x_4 - 128) + base[4]$$

Where $base$ is a table such that, being k the codeword length, $base[k] = (\sum_{j=1}^{k} 128^{j-1}) - 1$ indicates the rank in the vocabulary of the first word encoded with k bytes.

The pseudo-code for the *decode* algorithm is shown next.

```
Decode (base, x)
(1)  //input  x: the codeword to be decoded
(2)  //output i: the position of the decoded word in the ranked vocabulary
(3)  i ← 0;
(4)  k ← 1; // byte of the codeword
(5)  while x[k] < 128
(6)        i ← i × 128 + x[k];
(7)        k ← k + 1;
(8)  i ← i × 128 + x[k] − 128;
(9)  i ← i + base[k];
(10) return i;
```

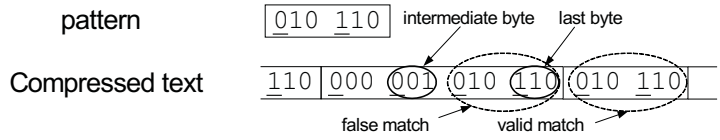


Figure 5.1: Searching End-Tagged Dense Code.

5.4 Searching End-Tagged Dense Code

As it was shown in Section 5.2, End-Tagged Dense Code uses a flag bit to mark the end of a codeword. For this reason, the concatenation of bytes from two consecutive codewords cannot produce a false matching. Therefore, as in Tagged Huffman, it is feasible to use a Boyer-Moore type searching which permits to skip some bytes of the codewords. However, it has to be taken into account that the End-Tagged Dense Code, is not a *suffix free code* (a codeword can be the suffix of another codeword), while Tagged Huffman is. Hence, each time a matching of the whole pattern occurs in the text, it is mandatory to check whether the byte preceding the first matched byte has either a value in the range $[2^{b-1}, 2^b - 1]$ (it is the last byte of the previous codeword) or a value in the range $[0, 2^{b-1} - 1]$ (it is not the last byte of the previous codeword but a part of the current codeword, which is longer than the pattern). If that value is greater than or equal to 2^{b-1}, then a valid match has to be reported. However, if it is smaller than 2^{b-1}, then the match is not valid since it does not correspond to the searched codeword, but to the suffix of a larger codeword, so the process continues. An example of how false matchings can be detected (using "bytes" of three bits) is presented in Figure 5.1.

This overhead in searches is negligible because checking the previous byte is only needed when a matching is detected, and it is not necessary during the search phase. As it is shown in Section 5.5.4, this small disadvantage with respect to Tagged Huffman (which is both a prefix and a suffix code) is compensated because the size of the compressed text is smaller in End-Tagged Dense Code than in Tagged Huffman.

5.5 Empirical results

The text collections described in Section 2.7 were compressed using Plain Huffman, End-Tagged Dense Code and Tagged Huffman. The spaceless word model [MNZBY00] was used to create the vocabulary. In the remainder of this section we

provide comparisons among the three compression techniques in both compression ratio, encoding and compression time, and decompression time.

5.5.1 Compression ratio

Table 5.5 shows the compression ratio obtained by the mentioned codes.

The size of the compressed vocabulary was excluded from the results. This size is negligible and similar in all cases, although a bit smaller in End-Tagged Dense Code because only the ordered list of words is needed.

The second column of the table contains the original size (in bytes) of the processed corpus, the third column indicates the number of words in the vocabulary, and the following three columns the compression ratio for each method. Finally, the last two columns show the differences (in percentage points) between End-Tagged Dense Code and Plain Huffman, and between End-Tagged Dense Code and Tagged Huffman respectively.

CORPUS	Original Size	n	PH	ETDC	TH	ETDC-PH	TH-ETDC
CALGARY	2,131,045	30,995	34.76	35.92	38.91	1.16	2.99
FT91	14,749,355	75,681	30.26	31.15	33.58	0.89	2.43
CR	51,085,545	117,713	29.21	30.10	32.43	0.89	2.33
FT92	175,449,235	284,892	30.49	31.31	34.08	0.82	2.77
ZIFF	185,220,215	237,622	31.83	32.72	35.33	0.89	2.61
FT93	197,586,294	291,427	30.61	31.49	34.30	0.88	2.81
FT94	203,783,923	295,018	30.57	31.46	34.28	0.89	2.82
AP	250,714,271	269,141	31.32	32.14	34.72	0.82	2.58
ALL_FT	591,568,807	577,352	30.73	31.56	34.16	0.83	2.60
ALL	1,080,719,883	886,190	32.05	32.88	35.60	0.83	2.72

Table 5.5: Comparison of compression ratios.

As it can be seen, Plain Huffman gets the best compression ratio (as expected since it is an optimal prefix code) and End-Tagged Dense Code always obtains better results than Tagged Huffman, with an improvement of up to 2.5 percentage points. In fact, End-Tagged Dense Code is worse than Plain Huffman only by less than 1 percentage point on average.

5.5.2 Encoding and compression times

Table 5.6 compares the code generation time for both End-Tagged Dense Code and Plain Huffman, that is, the time needed to assign a codeword to each word in the sorted vocabulary. A detailed description of how those processes were implemented

is presented in Section 6.6.2, where it is also shown how the implementation of our next compression method (the (s, c)-Dense Code) is done.

The first column in Table 5.6 indicates the corpus processed. Columns two and three show the number of words in the whole text and the number of words in the vocabulary respectively. The fourth and fifth columns give the encoding time (in milliseconds) for both End-Tagged Dense Code and Plain Huffman. Finally, the last column shows the gain (in percentage) of End-Tagged Dense Code with respect to Plain Huffman.

CORPUS	**#words**	**n**	**ETDC** (msec)	**Plain** (msec)	**DIFF** (%)
CALGARY	528,611	30,995	4.233	11.133	61.98
FT91	3,135,383	75,681	11.977	26.500	54.81
CR	10,230,907	117,713	21.053	49.833	57.75
FT92	36,803,204	284,892	52.397	129.817	59.64
ZIFF	40,866,492	237,622	44.373	105.900	58.10
FT93	42,063,804	291,427	52.813	133.350	60.40
FT94	43,335,126	295,018	52.980	134.367	60.57
AP	53,349,620	269,141	50.073	121.900	58.92
ALL_FT	124,971,944	577,352	103.727	260.800	60.23
ALL	229,596,845	886,190	165.417	402.875	58.94

Table 5.6: Code generation time comparison.

The results indicate that End-Tagged Dense Code is about 60% faster than Plain Huffman in the code generation phase. Note that the overall complexity is $O(n)$ in both methods. However, from a sorted vocabulary, End-Tagged Dense Code can immediately start code assignment, while Plain Huffman has to deal with the prior process of building a canonical Huffman tree. In the case of Tagged Huffman, results are similar to Plain Huffman but it is slightly slower because longer codewords are generated.

Note that vocabulary extraction, vocabulary sorting, and compression phases are common for both End-Tagged Dense Code and Plain Huffman. Unfortunately, those processes consume much more time than encoding. On the other hand, Plain Huffman has to output[2] less bytes than End-Tagged Dense Code. As a result, compression time and compression speed are more or less the same in both methods, as the next table shows.

Table 5.7 compares End-Tagged Dense Code with Plain Huffman in compression time. The third and fourth columns contain the user time (in seconds) needed to compress each corpus in our experimental framework. Compression speed, measured

[2]The *user time* does not take into account the time spent in I/O system calls. However, it includes the time needed to manage the operations with I/O buffers implied in both compression and decompression processes.

CORPUS	Size (bytes)	Compr. time (sec)		Compr. speed (Kbytes/sec)		
		ETDC	PH	ETDC	PH	DIFF(%)
CALGARY	2,131,045	0.393	0.415	5,417.91	5,135.05	5.508
FT91	14,749,355	2.482	2.500	5,943.33	5,899.74	0.739
CR	51,085,545	7.988	7.990	6,395.46	6,393.69	0.028
FT92	175,449,235	29.230	29.243	6,002.37	5,999.80	0.043
ZIFF	185,220,215	30.368	30.354	6,099.29	6,101.95	-0.044
FT93	197,586,294	32.783	32.915	6,027.12	6,002.93	0.403
FT94	203,783,923	33.763	33.874	6,035.81	6,015.98	0.330
AP	250,714,271	42.357	42.641	5,919.06	5,879.62	0.671
ALL_FT	591,568,807	100.469	99.889	5,888.10	5,922.28	-0.577
ALL	1,080,719,883	191.763	191.396	5,635.70	5,646.53	-0.192

Table 5.7: Compression speed comparison.

in Kbytes per second, is shown in columns five and six. The last column shows the gain (in percentage) of compression speed of End-Tagged Dense Code with respect to Plain Huffman. These results show that both techniques have similar compression speed because, although End-Tagged Dense Code outputs more data than Plain Huffman, its faster encoding process compensates the time needed to output longer codewords.

5.5.3 Decompression time

The decompression process is almost identical for Plain Huffman and End-Tagged Dense Code. The process starts by loading the words of the vocabulary into a vector V. In order to decode a codeword, Plain Huffman needs also to load two vectors: *base* and $first$ which implicitly represent a canonical Huffman tree. Next, the compressed text is read and each codeword is replaced by its corresponding uncompressed word. Given a codeword C, a simple decoding algorithm obtains the position i of the word in the vocabulary, such that $V[i]$ is the uncompressed word that corresponds to codeword C. Decompression takes $O(v)$ time, being v the size in bytes of the compressed text.

Plain Huffman and End-Tagged Dense Code were compared in decompression speed using all the text corpora in our experimental framework. Table 5.8 shows the results obtained. The size of the compressed text is shown in columns two and three. The next two columns present decompression time (in seconds). The sixth and the seventh columns show decompression speed (in Kbytes per second) and the last one shows the gain (in percentage) of decompression speed of End-Tagged Dense Code with respect to Plain Huffman. Decompression speed is almost identical (about 23 Mbytes/sec) in both cases.

CORPUS	Compressed Text Size (bytes)		Decompr. time (sec)		Decompress. speed (Kbytes/sec)		
	ETDC	PH	ETDC	PH	ETDC	PH	DIFF(%)
CALGARY	765,394	740,676	0.085	0.088	25,071.12	24,125.04	3.774
FT91	4,594,427	4,462,613	0.570	0.577	25,876.06	25,576.92	1.156
CR	15,374,717	14,923,066	1.926	1.903	26,530.29	26,851.80	-1.212
FT92	54,930,885	53,499,958	7.561	7.773	23,204.16	22,573.08	2.720
ZIFF	60,612,615	58,957,431	7.953	8.263	23,289.77	22,414.71	3.757
FT93	62,228,340	60,483,655	8.694	8.406	22,727.40	23,506.19	-3.427
FT94	64,112,806	62,303,279	8.463	8.636	24,080.82	23,596.34	2.012
AP	80,569,810	78,526,845	11.233	11.040	22,318.78	22,709.63	-1.751
ALL-FT	186,722,027	181,806,938	24.500	24.798	24,145.67	23,855.99	1.200
ALL	355,297,805	346,370,713	46.352	45.699	23,315.50	23,648.88	-1.430

Table 5.8: Decompression speed comparison.

5.5.4 Search time

As we introduced in Section 5.4, End-Tagged Dense Code is not a *suffix code*. This implies that each time a match ocurrs during the traversal of the compressed text, it has to be checked if it is a *valid match or not*. This is accomplished by inspecting the previous byte and checking whether it is the last byte of a codeword (that is, a value greater than $127 = 2^{b-1} - 1$).

Empirical results regarding direct search time in End-Tagged Dense Code and Tagged Huffman are presented in Table 5.9. The compressed text has been searched by first compressing the pattern and then performing a traversal through the compressed file to detect all the matches of the compressed pattern inside it. This traversal was carried out by applying *Horspool* pattern matching algorithm (Section 3.4.2).

Results shown in Table 5.9 compare the time needed to search for a single word pattern in three different corpora: FT91, AP, and ALL. We carefully chose those words in order to search for the most frequent words and for the least frequent words that are assigned 1, 2, and 3 bytes codewords when they are encoded with both Tagged Huffman and End-Tagged Dense Code. Note that in the case of End-Tagged Dense Code, all words were encoded with at most 3 bytes, while in Tagged Huffman, codewords of 4 bytes were needed to encode some words in corpora AP and ALL.

Table 5.9 contains three sub-tables, one sub-table corresponding to each corpus. They are organized as follows: The three first columns give the length of the codeword associated to the word being searched, the word itself and the number of occurrences of that word in the text. Columns four and five give the searching time (in seconds) for both techniques, and finally the last column gives the decrease of time (in percentage) of End-Tagged Dense Code with respect to Tagged Huffman.

Empirical results show that searching End-Tagged Dense Code for a k-byte

codeword is faster (up to 7%) than searching Tagged Huffman. Therefore, the extra comparison that is needed in End-Tagged Dense Code is compensated by its better compression ratio (which implies that a shorter file has to be traversed during searches).

FT91					
code length	**word**	**occurrences**	**ETDC (sec)**	**TH (sec)**	**DIFF (%)**
1	the	126,869	0.071	0.075	5.106
1	market	4,941	0.063	0.066	4.972
2	But	2,754	0.034	0.035	2.173
2	Maria	39	0.031	0.033	5.946
3	bells	7	0.023	0.024	1.634
3	citadels	1	0.022	0.023	3.489

AP					
code length	**word**	**occurrences**	**ETDC (sec)**	**TH (sec)**	**DIFF (%)**
1	the	1,970,841	1.236	1.308	5.498
1	percent	80,331	1.094	1.158	5.559
2	new	40,347	0.572	0.615	6.927
2	Platinum	634	0.567	0.598	5.282
3	Chang	143	0.401	0.407	1.304
3	Lectures	5	0.369	0.398	7.215

ALL					
code length	**word**	**occurrences**	**ETDC (sec)**	**TH (sec)**	**DIFF (%)**
1	the	8,205,778	5.452	5.759	5.331
1	were	356,144	5.048	5.182	2.596
2	over	202,706	2.737	2.859	4.279
2	predecessor	2,775	2.666	2.736	2.558
3	resilience	612	1.779	1.858	4.257
3	behooves	22	1.667	1.701	2.009

Table 5.9: Searching time comparison.

It is also interesting to note that, since a Boyer-Moore-type searching is used, longer patterns are found faster. As a result, a word that is encoded with 4 bytes by Tagged Huffman is found much faster than the same word (which is encoded with 3 bytes) in End-Tagged Dense Code. Table 5.10 shows these results. For example, in corpus ALL, 779,375 words (87.95%) are encoded with 4 bytes in Tagged Huffman.

Table 5.11 shows the results of searching for random words appearing at least twice in each corpus. We took 10,000 random samples (words) from the vocabulary of each corpus and then we searched for them. Average-search-time ($\overline{time}$) and standard deviation (σ) are given for each compression technique. The second and third columns in that table, show the average-time and the standard deviation for End-Tagged Dense Code. Average-time and standard deviation for Tagged Huffman are shown in columns 4 and 5. Finally, the last column shows the difference of search

AP					
code length	**word**	**occurrences**	**ETDC (sec)**	**TH (sec)**	**DIFF (%)**
(*) 3-4	Carelly	5	0.367	0.300	-22.474
(*) 3-4	Britany	1	0.370	0.298	-24.336

ALL					
code length	**word**	**occurrences**	**ETDC (sec)**	**TH (sec)**	**DIFF (%)**
(*) 3-4	predate	22	1.677	1.389	-20.810
(*) 3-4	Fezzani	1	1.641	1.387	-18.320

(∗)these codewords have 3 bytes in ETDC and 4 bytes in TH.

Table 5.10: Searching time comparison.

time, in percentage points, between Tagged Huffman and End-Tagged Dense Code.

In general, End-Tagged Dense Code obtains the best results when searching for random single-word patterns. Only in the largest texts, Tagged Huffman obtains the best search time. As we have already explained, this is because the number of long codewords in Tagged Huffman is larger than in End-Tagged Dense Code. Since the longer the codeword, the faster the search using Boyer-Moore type algorithms, a trade-off between compression ratio and search speed exists.

Even though Tagged Huffman is faster in searches for random codewords in large texts, End-Tagged Dense Code is always faster when codewords of a given length are searched.

CORPUS	**ETDC**		**TH**		**DIFF** (%)
	time	σ	time	σ	TH-ETDC
CALGARY	0.005	0.017	0.005	0.020	0.000
FT91	0.024	0.006	0.024	0.005	0.000
CR	0.073	0.016	0.077	0.012	5.479
FT92	0.257	0.044	0.267	0.046	3.891
ZIFF	0.283	0.047	0.292	0.052	3.180
FT93	0.291	0.045	0.299	0.052	2.749
FT94	0.300	0.047	0.306	0.059	2.000
AP	0.382	0.066	0.380	0.048	-0.524
ALL_FT	0.867	0.101	0.760	0.141	-12.341
ALL	1.650	0.195	1.390	0.250	-15.758

Table 5.11: Searching for random patterns: time comparison.

5.6 Summary

End-Tagged Dense Code, a simple and fast technique for compressing natural language texts databases, was presented. Its compression scheme is not based on Huffman at all, but it generates *prefix codes* by using a tag bit that indicates whether each byte is the last of the codeword or not. Being a Tagged code, direct access, searching the compressed text, and direct decompression are possible.

Empirical results show that the compression obtained is very good. It achieves less than one percentage point of compression ratio overhead with respect to Plain Huffman, and improves that of Tagged Huffman by about 2.5 percentage points. Moreover, compression speed is similar in both End-Tagged Dense Code and Plain Huffman, but encoding is faster (about 60%), because the encoding algorithm is simpler. In decompression, End-Tagged Dense Code is slightly faster than Plain Huffman. Being faster that Plain Huffman guarantees that End-Tagged Dense Code is also faster than Tagged Huffman since both encoding time and compression ratio are better in Plain Huffman than in Tagged Huffman [MNZBY00].

With respect to searches, we compared End-Tagged Dense Code with Tagged Huffman because Plain Huffman permits neither direct search nor direct access. We found that pattern matching of k-byte codewords over End-Tagged Dense Code is around 5 percentage points faster than in Tagged Huffman. Only random searches in large text collections are faster in Tagged Huffman due to its worse compression ratio. The worse the compression ratio, the longer the search patterns, and consequently, the faster the search.

6

(s, c)-Dense Code

The second contribution of this thesis is presented in this chapter. It consists of a new word-based byte-oriented statistical *two-pass* technique called (s, c)-Dense Code, which generalizes the End-Tagged Dense Code technique described in the previous chapter.

This chapter is structured as follows: First, the key idea and the motivation of the (s, c)-Dense Code are shown in Section 6.1. In Section 6.2, the new technique is defined and described. Next, procedures to obtain optimal s and c parameters are presented, encoding and decoding processes are explained, and their pseudo-code is given. Section 6.5 is focused on how direct searches can be performed over this code. In Section 6.6, empirical results comparing (s, c)-Dense Code against End-Tagged Dense Code, Plain Huffman and Tagged Huffman are also shown. The chapter ends with some conclusions.

6.1 Motivation

End-Tagged Dense Code has been described in Chapter 5. This technique uses a semi-static statistical word-based model as the Plain Huffman and Tagged Huffman codes [MNZBY00] do, but it is not based on Huffman at all.

In Section 5.5 it is shown that End-Tagged Dense Code improves the Tagged Huffman compression ratio by more than 2.5 percentage points. However, its difference with respect to Plain Huffman is still about 1 percentage point.

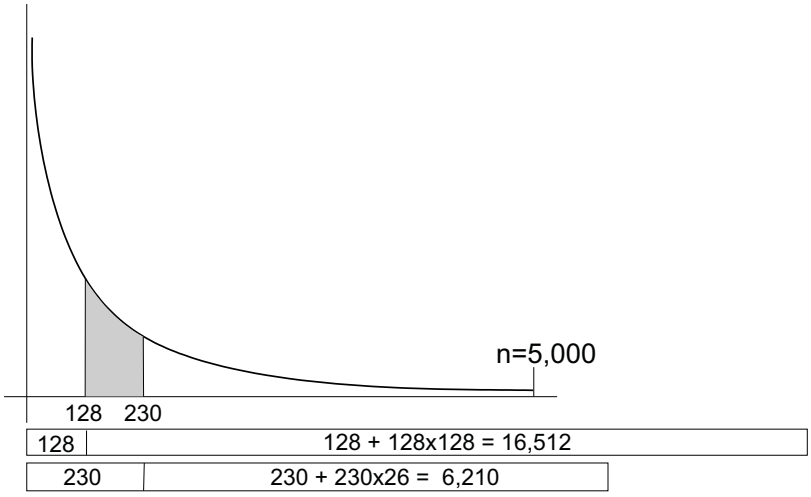


Figure 6.1: 128 versus 230 *stoppers* with a vocabulary of $5,000$ words.

As it is shown in Chapter 5, End-Tagged Dense Code uses 2^{b-1} digits, from 0 to $2^{b-1} - 1$, for the bytes that do not end a codeword, and it uses the other 2^{b-1} digits, from 2^{b-1} to $2^b - 1$, for the last byte of the codeword. Let us call *continuers* the former digits and *stoppers* the latter. The question that arises now is whether that proportion between the number of continuers and stoppers is the optimal one; that is, for a given corpus with a specific word frequency distribution, we want to know the optimal number of continuers and stoppers. It will probably be different than 2^{b-1}. This idea has been previously pointed out in [RTT02], and it is the basic difference between End-Tagged Dense Code and (s, c)-Dense Code, as it will be explained later. In [RTT02], an encoding scheme using stoppers and continuers is presented on a character-based source alphabet, but the goal of those authors was to have a code where searches could be performed very efficiently. Their idea was to create a code where each codeword could be split into two parts in such a way that searches could be performed using only one part of the codewords.

Example 6.1 illustrates the advantages of using a variable rather than a fixed number of stoppers and continuers.

Example 6.1 Let us suppose that 5,000 distinct words compose the vocabulary of the text to compress. Assume also that bytes are used to form the codewords, so $b = 8$.

If End-Tagged Dense Code is used, that is, if the number of stoppers and

continuers is $2^7 = 128$, there will be 128 codewords of one byte, and the rest of the words would have codewords of two bytes, since $128 + 128^2 = 16,512$. That is, $16,512$ is the number of words that can be encoded with codewords of one or two bytes. Therefore, there would be $16,512 - 5,000 = 11,512$ unused codewords of two bytes.

If the number of stoppers chosen is 230 (so the number of continuers is $256-230 = 26$), then $230+230\times26 = 6,210$ words can be encoded with codewords of only one or two bytes. Therefore the whole $5,000$ words can be assigned codewords of 1 or 2 bytes in the following way: the 230 most frequent words are assigned one-byte codewords and the remainder $5,000 - 230 = 4,770$ words are assigned two-byte codewords.

It can be seen that words from 1 to 128 and words ranked from 231 to $5,000$ are assigned codewords of the same length in both schemes. However words from 129 to 230 are assigned shorter codewords when using 230 stoppers instead of only 128. Figure 6.1 describes this scenario. □

As a result, it seems appropriate to adapt the number of stoppers and continuers to:

- The size of the vocabulary (n). It is possible to maximize the available number of short codewords depending on n.
- The word frequency distribution of the vocabulary. Intuitively, if that distribution presents a very steep slope (that is, if it is very biased), it can be desirable to increase the number of words that can be encoded with short codewords (so a high number of stoppers should be chosen). However, this will imply that the less frequent words are encoded with longer codewords, which does not matter since the gain in the most frequent words compensates the loss of compression in the least frequent words. When that distribution is more uniform, it is preferable to reduce the longest-codeword length, in order to avoid losing compression in the least frequent words.

6.2 (s, c)-Dense Code

We define (s, c) stop-cont codes as follows.

Definition 6.1 *Given source symbols with probabilities* $\{p_i\}_{0\leq i<n}$ *an* (s, c) *stop-cont code (where* c *and* s *are integers larger than zero) assigns to each source symbol*

i a unique target code formed by a sequence of zero or more digits in base c added to s (that is, from s to $s + c - 1$). That sequence is terminated by a base-s digit (between 0 and $s - 1$).

It should be clear that a stop-cont coding is just a base-c numerical representation, yet adding s to each digit, with the exception that the last digit is in base s, between 0 and $s-1$. Continuers are digits between s and $s+c-1$ and stoppers are digits between 0 and $s - 1$. The next property clearly follows.

Property 6.1 *Any (s, c) stop-cont code is a prefix code.*

Proof If one code were a prefix of the other, since the shorter code must have a final digit of value lower than s, then the longer code must have an intermediate digit which is not in base c plus s. This is a contradiction. □

Among all the possible (s, c) stop-cont codes for a given probability distribution, the *dense code* is the one that minimizes the average codeword length. This is because a dense code uses all the possible combinations of bits in each byte. That is, codes can be assigned sequentially to the ranked symbols.

Definition 6.2 *Given source symbols with* decreasing *probabilities $\{p_i\}_{0 \le i < n}$, the corresponding (s, c)-Dense Code ((s, c)-DC) is an (s, c) stop-cont code where the codewords are assigned as follows: Let k be the number of bytes in each codeword, which is always ≥ 1, then k will be such that*

$$s\frac{c^{k-1} - 1}{c - 1} \leq i < s\frac{c^k - 1}{c - 1}$$

Thus, the code corresponding to source symbol i is formed by $k-1$ digits in base c added to s and a final base-s digit. If $k = 1$ then the code is simply the stopper i. Otherwise the code is formed by the number $\lfloor x/s \rfloor$ written in base c, and adding s to each digit (they are base-c digits which are then added to s), followed by x mod s, where $x = i - \frac{sc^{k-1} - s}{c-1}$.

That is, using symbols of 8 bits ($b = 8$), the encoding process can be described as follows:

- One-byte codewords from 0 to $s - 1$ are given to the first s words in the vocabulary.

- Words ranked from s to $s+sc-1$ are assigned sequentially two-byte codewords. The first byte of each codeword has a value in the range $[s, s + c - 1]$ and the second in range $[0, s - 1]$.
- Words from $s + sc$ to $s + sc + sc^2 - 1$ are assigned tree-byte codewords, and so on.

Word rank	codeword assigned	# bytes	# words
0	[0]	1	
1	[1]	1	
2	[2]	1	s
...	...	...	
$s-1$	[s-1]	1	
s	[s][0]	2	
$s+1$	[s][1]	2	
$s+2$	[s][2]	2	
...	...	...	
$s+s-1$	[s][s-1]	2	sc
$s+s$	[s+1][0]	2	
$s+s+1$	[s+1][1]	2	
...	...	...	
$s+sc-1$	[s+c-1][s-1]	2	
$s+sc$	[s][s][0]	3	
$s+sc+1$	[s][s][1]	3	
$s+sc+2$	[s][s][2]	3	sc^2
...	...	...	
$s+sc+sc^2-1$	[s+c-1][s+c-1][s-1]	3	
...	...	...	

Table 6.1: Code assignment in (s, c)-Dense Code.

Table 6.1 summarizes this process. Next, we provide another example of how codewords are assigned.

Example 6.2 The codes assigned to symbols $i \in 0 \ldots 15$ by a (2,3)-DC are as follows: ⟨0⟩, ⟨1⟩, ⟨2,0⟩, ⟨2,1⟩, ⟨3,0⟩, ⟨3,1⟩, ⟨4,0⟩, ⟨4,1⟩, ⟨2,2,0⟩, ⟨2,2,1⟩, ⟨2,3,0⟩, ⟨2,3,1⟩, ⟨2,4,0⟩, ⟨2,4,1⟩, ⟨3,2,0⟩ and ⟨3,2,1⟩. □

Notice that the code does not depend on the exact symbol probabilities, but only on their ordering by frequency. We now prove that the dense coding is an optimal stop-cont coding.

Property 6.2 *The average length of an (s, c)-dense code is minimal with respect to any other (s, c) stop-cont code.*

Proof Let us consider an arbitrary (s, c) stop-cont code, and let us write all the possible codewords in numerical order, as in Example 6.2, together with the symbol they encode, if any. Then it is clear that (i) any unused code in the middle could be used to represent the source symbol with longest codeword, hence a compact assignment of target symbols is optimal; and (ii) if a less probable symbol with a shorter code is swapped with a more probable symbol with a longer code then the average code length decreases, and hence sorting the symbols by decreasing frequency is optimal. □

Since sc^{k-1} different codewords can be coded using k digits, let us call

$$W_k^s = \sum_{j=1}^{k} sc^{j-1} = s\frac{c^k - 1}{c - 1} \tag{6.1}$$

(where $W_0^s = 0$) the number of source symbols that can be coded with up to k digits. Let us also call

$$f_k^s = \sum_{j=W_{k-1}^s+1}^{W_k^s} p_j \tag{6.2}$$

the sum of probabilities of source symbols coded with k digits by an (s, c)-DC.

Then, the average codeword length, $L_d(s, c)$, for the (s, c)-DC is

$$\begin{aligned} L_d(s,c) &= \sum_{k=1}^{K^s} k f_k^s = \sum_{k=1}^{K^s} k \sum_{j=W_{k-1}^s+1}^{W_k^s} p_j \\ &= 1 + \sum_{k=1}^{K^s-1} k \sum_{j=W_k^s+1}^{W_{k+1}^s} p_j = 1 + \sum_{k=1}^{K^s-1} \sum_{j=W_k^s+1}^{W_{K^s}^s} p_j \end{aligned} \tag{6.3}$$

where $K^x = \left\lceil \log_{(2^b-x)} \left(1 + \frac{n(2^b-x-1)}{x}\right) \right\rceil$, and n is the number of symbols in the vocabulary.

It is clear from Definition 6.2 that the End-Tagged Dense Code is a $(2^{b-1},2^{b-1})$-DC and therefore *(s,c)*-DC can be seen as a generalization of the End-Tagged Dense Code where s and c are adjusted to optimize the compression for the distribution of frequencies and the size of the vocabulary.

Moreover, recall from Chapter 5 that $(2^{b-1},2^{b-1})$-DC is more efficient than Tagged Huffman over b bits. This is because Tagged Huffman is essentially a $(2^{b-1},2^{b-1})$ *(non dense) stop-cont* code, while the End-Tagged Dense Code is a $(2^{b-1},2^{b-1})$-Dense Code.

Rank	Word	Freq	PH	(6,2)-DC	ETDC	TH	Freq × bytes			
							PH	(6,2)-DC	ETDC	TH
0	A	0.200	[0]	[0]	[4]	[4]	0.20	0.20	0.20	0.20
1	B	0.200	[1]	[1]	[5]	[5]	0.20	0.20	0.20	0.20
2	C	0.150	[2]	[2]	[6]	[6]	0.15	0.15	0.15	0.15
3	D	0.150	[3]	[3]	[7]	[7][0]	0.15	0.15	0.15	0.30
4	E	0.140	[4]	[4]	[0][4]	[7][1]	0.14	0.14	0.28	0.28
5	F	0.090	[5]	[5]	[0][5]	[7][2]	0.09	0.09	0.18	0.18
6	G	0.040	[6]	[6][0]	[0][6]	[7][3][0]	0.04	0.08	0.08	0.12
7	H	0.020	[7][0]	[6][1]	[0][7]	[7][3][1]	0.04	0.04	0.04	0.06
8	I	0.005	[7][1]	[6][2]	[1][4]	[7][3][2]	0.01	0.01	0.01	0.015
9	J	0.005	[7][2]	[6][3]	[1][5]	[7][3][3]	0.01	0.01	0.01	0.015
						average codeword length	**1.03**	**1.07**	**1.30**	**1.52**

Table 6.2: Comparative example among compression methods, for b=3.

Example 6.3 Table 6.2 shows the codewords assigned to a small set of words ordered by frequency when using Plain Huffman (P.H.), *(6,2)*-DC, End-Tagged Dense Code (ETDC) which is a *(4,4)*-DC, and Tagged Huffman (TH). Digits of three bits (instead of bytes) are used for simplicity (b=*3*), and therefore $s + c = 8$. The last four columns present the products of the number of bytes by the frequency for each word, and its sum (the average codeword length) is shown in the last row.

It is easy to see that, for this example, Plain Huffman and the (6,2)-Dense Code are better than the (4,4)-Dense Code (ETDC) and therefore they are also better than Tagged Huffman. Notice that (6,2)-Dense Code is better than (4,4)-Dense Code because it takes advantage of the distribution of frequencies and of the number of words in the vocabulary. However, the values (6,2) for s and c are not the optimal ones since a (7,1)-Dense Code obtains, in this example, an optimal compressed text having the same result than Plain Huffman. □

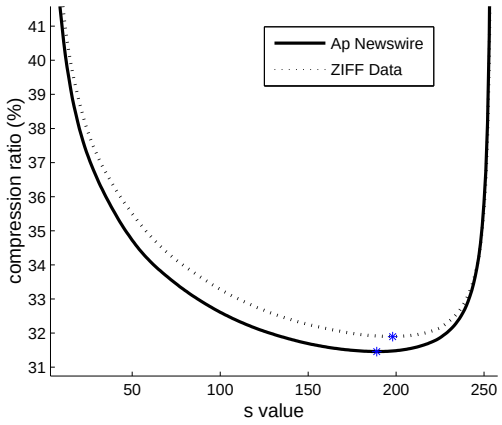
The problem now consists of finding the s and c values (assuming a fixed b where $2^b = s+c$) that minimize the size of the compressed text for a specific word frequency distribution.

6.3 Optimal s and c values

The key advantage of this method with respect to End-Tagged Dense Code is the ability to use the optimal s and c values. In all the real text corpora used in our experiments, the size of the compressed text, as a function of s, has only one local minimum. See Figures 6.2 and 6.3, where optimal s values are shown for some real corpora, as well as the curves where it can be seen that a unique minimum exits.

In Figure 6.2, the size of the compressed texts and the compression ratios are shown as a function of the values of s, for ZIFF and AP corpora. As it is also shown

in the table of Figure 6.2, the optimal s value for ZIFF corpus is 198, while for the AP corpus the best compression ratio is achieved with $s = 189$. That table shows sizes and compression ratios when values of s close to the optimum are used over these two corpus.



s value	Ap Newswire Corpus		ZIFF Corpus	
	ratio(%)	size(bytes)	ratio(%)	size(bytes)
186	31.5927	79,207,309	32.0433	59,350,593
187	31.5917	79,204,745	32.0391	59,342,810
188	31.5910	79,202,982	32.0355	59,336,069
189	**31.5906**	**79,202,053**	32.0321	59,329,848
190	31.5907	79,202,351	32.0290	59,324,149
191	31.5912	79,203,574	32.0263	59,319,106
192	31.5921	79,205,733	32.0239	59,314,667
195	31.5974	79,219,144	32.0188	59,305,267
196	31.6002	79,226,218	32.0179	59,303,599
197	31.6036	79,234,671	32.0174	59,302,677
198	31.6074	79,244,243	**32.0174**	**59,302,609**
199	31.6117	79,254,929	32.0178	59,303,324
200	31.6166	79,267,125	32.0186	59,304,790
201	31.6220	79,280,683	32.0198	59,307,119

Figure 6.2: Compressed text sizes and compression ratios for different s values.

Note that, when s is very small, the number of high frequency words encoded with one byte is also very small (s words are encoded with just one byte) but in this case, c is large and therefore words with low frequency will be encoded with few bytes: sc words will be encoded with two bytes, sc^2 words will be encoded with 3 bytes, sc^3 with 4 bytes, and so on.

It is clear that, as s grows, more high frequency words will be encoded with one byte, so we improve the compression of those words. But at the same time, as s grows, more low frequency words will need more bytes to be encoded, so we lose compression in those words.

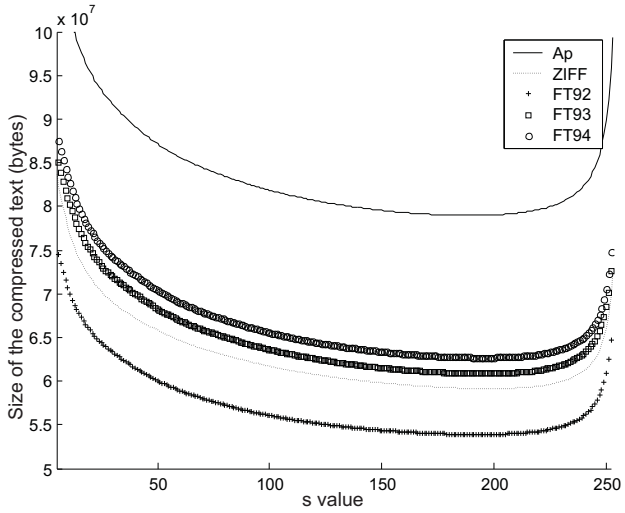


Figure 6.3: Size of the compressed text for different s values.

At some point, the compression lost in the last words is larger than the compression gained in words at the beginning, and therefore the global compression ratio decreases. That point gives us the optimal s value. It is easy to see in Figure 6.3 that, around of the optimal value, the compression is relatively insensitive to the exact value of s. This fact causes the smooth bottom part of the curve.

A binary search algorithm, which computes the best s and c values, takes advantage of this property. It is not necessary to check all the values of s because the shape of the distribution of compression ratios as a function of s is known. Therefore the search is led towards the area where compression ratio is best.

However, the property of the existence of a unique minimum does not always hold. We have produced artificial distributions where more than one local minima exist. A distribution with 2 local minima is given in Example 6.4. However, although a sequential search for the optimal s and c values would be theoretically necessary, a binary search will, in real cases, find the best s and c values. In all the real natural language corpora checked, only one local minimum appeared (we explain in Section 6.3.1 the reasons why this occurs). On the other hand, the bottom of the curve is very smooth, which means that even if an s value that is not exactly the optimal one is chosen, the loss of compression ratio will be very small.

Example 6.4 Consider a text with $N = 50,000$ words, and $n = 5,000$ distinct

s value	average codeword length	compressed text size (bytes)
7	$(1 \times 0.4014 \times 1) + (1 \times 0.044 \times 6) + (2 \times 0.044 \times 3)$ $+(2 \times 0.0001 \times 50) + (2 \times 0.00004 \times 10)$ $+(3 \times 0.00004 \times 567) + (4 \times 0.00004 \times 4,363) = 1.70632$	85,316
8	$(1 \times 0.4014 \times 1) + (1 \times 0.044 \times 7) + (2 \times 0.044 \times 2)$ $+(2 \times 0.0001 \times 50) + (2 \times 0.00004 \times 12)$ $+(3 \times 0.00004 \times 512) + (4 \times 0.00004 \times 4,096) =$ $+(5 \times 0.00004 \times 320) =$ **1.67716**	**83,858**
9	$(1 \times 0.4014 \times 1) + (1 \times 0.044 \times 8) + (2 \times 0.044 \times 1)$ $+(2 \times 0.0001 \times 50) + (2 \times 0.00004 \times 12)$ $+(3 \times 0.00004 \times 441) + (4 \times 0.00004 \times 3,087) =$ $+(5 \times 0.00004 \times 1,400) = 1.67920$	83,960
10	$(1 \times 0.4014 \times 1) + (1 \times 0.044 \times 9) + (2 \times 0.0001 \times 50)$ $+(2 \times 0.00004 \times 10) + (3 \times 0.00004 \times 360)$ $+(4 \times 0.00004 \times 2,160) = +(5 \times 0.00004 \times 2,410)$ $=$ **1.67900**	**83,950**
11	$(1 \times 0.4014 \times 1) + (1 \times 0.044 \times 9) + (1 \times 0.0001 \times 1)$ $+(2 \times 0.0001 \times 49) + (2 \times 0.00004 \times 6)$ $+(3 \times 0.00004 \times 275) + (4 \times 0.00004 \times 1,375) =$ $+(5 \times 0.00004 \times 3,284) = 1.71758$	85,879

Table 6.3: Size of compressed text for an artificial distribution.

words. An artificial distribution of the probability of occurrence p_i for all words i, $i \in 1..n$ in the text is defined as follows:

$$p_i = \begin{cases} 0.4014 & \text{if } i = 0 \\ 0.044 & \text{if } i \in [1..9] \\ 0.0001 & \text{if } i \in [10..59] \\ 0.00004 & \text{if } i \in [60..4999] \end{cases}$$

If the text is compressed using (s, c)-Dense Code and assuming that $b = 4$ (therefore, $s + c = 2^b = 16$), the distribution of the size of the compressed text depending on the value of s used to encode words has two local minima. The first minimum occurs when $s = c = 8$ and the second one when $s = 10$ and $c = 6$. This situation can be observed in Table 6.3. □

6.3.1 Feasibility of using binary search in natural language corpora

To understand why only a local minimum was found in all our experiments it is necessary to remember some properties of natural language text.

From the Heaps' law (see Section 2.4.1) it is easy to conclude that the number of different words that can be found in a natural language text grows very slowly when very large texts are used. In fact, the ALL corpus, a very large text (1,080,719,883 bytes), contains only 886,190 different words.

The other property, which is leading us to find just one minimum value of s, is derived from the fact that we always use bytes ($b = 8$) as basic symbols of the

s	W_1^s	W_2^s	W_3^s	W_4^s	W_5^s	W_6^s
1	1	256	65,281	16,711,681	4.E+09	1.E+17
10	10	2,470	607,630	150,082,150	4.E+10	9.E+17
20	20	4,740	1,118,660	265,117,700	6.E+10	1.E+18
30	30	6,810	1,539,090	349,366,650	8.E+10	2.E+18
40	40	8,680	1,874,920	406,849,000	9.E+10	**2.E+18**
50	50	10,350	2,132,150	441,344,750	**9.E+10**	2.E+18
60	60	11,820	2,316,780	**456,393,900**	9.E+10	2.E+18
70	70	13,090	2,434,810	455,296,450	8.E+10	2.E+18
80	80	14,160	2,492,240	441,112,400	8.E+10	1.E+18
90	90	15,030	**2,495,070**	416,661,750	7.E+10	1.E+18
100	100	15,700	2,449,300	384,524,500	6.E+10	9.E+17
110	110	16,170	2,360,930	347,040,650	5.E+10	7.E+17
120	120	16,440	2,235,960	306,310,200	4.E+10	6.E+17
127	127	16,510	2,129,917	276,872,827	4.E+10	5.E+17
128	128	**16,512**	2,113,664	272,646,272	3.E+10	4.E+17
129	129	16,512	2,097,153	268,419,201	3.E+10	4.E+17
130	130	16,510	2,080,390	264,193,150	3.E+10	4.E+17
140	140	16,380	1,900,220	222,309,500	3.E+10	3.E+17
150	150	16,050	1,701,450	182,039,250	2.E+10	2.E+17
160	160	15,520	1,490,080	144,522,400	1.E+10	1.E+17
170	170	14,790	1,272,110	110,658,950	1.E+10	8.E+16
180	180	13,860	1,053,540	81,108,900	6.E+09	5.E+16
190	190	12,730	840,370	56,292,250	4.E+09	2.E+16
200	200	11,400	638,600	36,389,000	2.E+09	1.E+16
210	210	9,870	454,230	21,339,150	1.E+09	5.E+10
220	220	8,140	293,260	10,842,700	390,907,660	1.E+10
230	230	6,210	161,690	4,359,650	113,662,090	3.E+09
240	240	4,080	65,520	1,110,000	17,883,120	286,314,480
250	250	1,750	10,750	73,750	460,750	2,791,750
255	**255**	510	765	1,275	2,040	3,060

Table 6.4: Values of W_k^s for $k \in [1..6]$.

codewords. See Table 6.4 to understand how the W_i^s evolve, assuming that $b = 8$.

The maximum number of words that can be encoded with 6 bytes is found when the value of s is around 40. In the same way, the maximum number of words that can be encoded with 5, 4, and 3 bytes is reached when the value of s is respectively around of 50, 60 and 90. Finally, the value of s that maximizes the number of words encoded with 2 bytes is $s = c = 128$, but the number of words encoded with just one byte grows when s is increased.

Notice that compression clearly improves, even if a huge vocabulary of 2 million words is considered, when s increases from $s = 1$ until $s = 128$. Only vocabularies with more than 2.5 million words can lose compression if s grows from $s = 90$ up to $s = 120$. This happens because those words that can be encoded with 3-byte codewords for $s = 90$, would need 4-byte codewords when s increases. However, as it has been already pointed out, we never obtained a vocabulary with more than $886,190$ words in all the real texts used, and that number of words is encoded with just 3 bytes with any $s \leq 187$.

Therefore, in our experiments the space trade-off depends on the sum of the probability of the words encoded with only 1 byte, against the sum of the probability of words encoded with 2 bytes. The remaining words were always encoded with 3 bytes.

The size of the compressed text for two consecutive values of s, let us call them T_s and T_{s+1}, are respectively:

$$T_s = 1 \sum_{i=0}^{W_1^s-1} p_i + 2 \sum_{W_1^s}^{W_2^s-1} p_i + 3 \sum_{i=W_2^s}^{W_3^s-1} p_i$$

$$T_{s+1} = 1 \sum_{i=0}^{W_1^{s+1}-1} p_i + 2 \sum_{i=W_1^{s+1}}^{W_2^{s+1}-1} p_i + 3 \sum_{i=W_2^{s+1}}^{W_3^{s+1}-1} p_i$$

Two different situations happen depending on whether $s > c$ or $s \leq c$. When $s < c$ the size of T_s is always greater than the size of T_{s+1} because the number of words that are encoded with both one and two bytes grows when s increases. Therefore as the value of s is increased, compression is improved until the value $s = c = 128$ is reached. Of course, this holds because after $s = 15$ all the words in the vocabulary are encoded with up to 3 bytes, hence variations in the number of words that could be encoded with 4, 5 or more bytes do not affect the compression achieved.

For s values beyond $s = c$ $(s > c)$, compression increases when the value $s + 1$ is used instead of s iff:

$$\sum_{i=W_1^s}^{W_1^{s+1}-1} p_i > \sum_{i=W_2^{s+1}}^{W_2^s-1} p_i \quad \text{that is,}$$

$$p_s > \sum_{i=(s+1)+(s+1)(c-1)}^{s+sc-1} p_i$$

It is clear that, for each successive value of s, p_s decreases while $\sum_{i=(s+1)+(s+1)(c-1)}^{s+sc-1} p_i = \sum_{sc+c}^{sc+s-1} p_i$ grows, since words are decreasingly sorted by frequency, and each interval $[W_2^{s+1},\ W_2^s - 1]$ is larger than the former. That is:

$$(W_2^s - 1) - W_2^{s+1} > (W_2^{s-1} - 1) - W_2^s$$

$$(s + sc - 1) - ((s + 1) + (s + 1)(c - 1)) > ((s - 1) + (s - 1)(c + 1) - 1) - s - sc$$

$$1 > -1$$

As a consequence, once s reaches a value such that $p_s \leq \sum_{i=W_2^{s+1}}^{W_2^s-1} p_i$, successive values of s will also produce a loss of compression. Such loss of compression will be bigger in each successive step.

To sum up, we can affirm that a unique optimal value of s will be found in practice, even if a huge natural language text is compressed. Moreover, that optimal value of s will be larger than 128, that is, $s > c$.

6.3.2 Algorithm to find the optimal s and c values

This section presents both the binary and sequential algorithms developed to obtain the optimal s and c values for a given vocabulary.

Both algorithms *BinaryFindBestS()* and *SequentialFindBestS()* need to compute the size (in bytes) of the compressed text for any given s and c values, in order to choose the best ones. This size is computed by another algorithm called *computeSizeS()*.

In turn, *computeSizeS()* uses a list of accumulated frequencies $acc[]$ previously computed to efficiently obtain the size of the compressed text. The size of the compressed text can be computed for any given s and c values, as follows:

$$\begin{aligned} size_{(bytes)} = acc[s] \quad &+ \quad \sum_{k=2}^{K^s-1} k\left(acc[W_k^s] - acc[W_{k-1}^s]\right) + \\ &+ \quad K^s\left(acc[n] - acc[W_{K^s-1}^s]\right) \end{aligned}$$

where $K^x = \left\lceil \log_{(2^b-x)}\left(1 + \frac{n(2^b-x-1)}{x}\right)\right\rceil$. The formula for $size$ can be reexpressed as follows:

$$\begin{aligned} size &= acc[s] + \sum_{k=2}^{K^s-1} k\left(acc[W_k^s] - acc[W_{k-1}^s]\right) + \\ &\quad K^s acc[n] - K^s acc[W_{K^s-1}^s] \\ &= W_1^s + 2acc[W_2^s] - 2acc[W_1^s] + 3acc[W_3^s] - 3acc[W_2^s] + \cdots + \\ &\quad (K^s-1)acc[W_{K^s-1}^s] - (K^s-1)acc[W_{K^s-2}^s] + \\ &\quad K^s acc[n] - K^s acc[W_{K^s-1}^s] \\ &= -acc[W_1^s] - acc[W_2^s] - \cdots - acc[W_{K^s-1}^s] + K^s acc[n] \\ &= acc[n] + \sum_{k=1}^{K^s-1} (acc[n] - acc[W_k^s]) \end{aligned}$$

The pseudo-code of *computeSizeS()* is as follows:

```
computeSizeS (s, c, acc)
(1)  //inputs: s, c and acc, the vector of accumulated frequencies
(2)  //output: the length of the compressed text using s and c
(3)  k ← 1; n ← number ofpositions in vector 'acc';
(4)  total ← acc[n];
(5)  Left ← min(s, n);
(6)  while Left < n
(7)        total ← total + (acc[n] − acc[Left]);
(8)        Left ← Left + sc^k;
(9)        k ← k + 1;
(10) return total;
```

Notice that the complexity of computing the size of the compressed text for a specific value of s is $O(\log_c n)$, except for $c = 1$, in which case it is $O(n/s) = O(n/2^b)$.

Sequential search

Sequentially searching the best s and c values consists of computing the size of the compressed text for each possible s value and then choosing the s value that minimizes the compressed text size.

```
SequentialFindBestS (b, acc)
(1)  //inputs: b value (2^b = c + s) and acc, the vector of accumulated frequencies
(2)  //output: The best s and c values
(3)  sizeBestS ← ∞;
(4)  for i = 1 to 2^b − 1
(5)        sizeS ← computeSizeS(i, 2^b − i, acc);
(6)        if sizeS < sizeBestS then
(7)              bestS ← i;
(8)              sizeBestS ← sizeS;
(9)  bestC ← 2^b − bestS;
(10) return bestS, bestC;
```

Since this algorithm calls *computeSizeS()* for each $s \in [1..2^b - 1]$, the cost of sequential search is:

$$O\left(\frac{n}{2^b} + \sum_{i=2}^{2^b-1} \log_i n\right) = O\left(\frac{n}{2^b} + \log_2 n \sum_{i=2}^{2^b-1} \frac{1}{\log_2 i}\right) = O\left(\frac{n}{2^b} + 2^b \log_2 n\right)$$

Note that $\int_2^{2^b} \frac{dx}{\log_2 x} \leq \sum_{i=2}^{2^b-1} \frac{1}{\log_2 i} \leq \int_1^{2^b-1} \frac{dx}{\log_2 x}$. Since $\frac{1}{\log x} = \Omega(\frac{1}{x^\epsilon}), \forall \epsilon > 0$,

solving both integrals we obtain: $\int_2^{2^b} \frac{dx}{\log_2 x} = \Theta(2^{b(1-\epsilon)})$ and $\int_1^{2^b-1} \frac{dx}{\log_2 x} = O(2^{b(1-\epsilon')})$. Therefore $\sum_{i=2}^{2^b-1} \frac{1}{\log_2 i} = \Omega(2^{b(1-\epsilon)})$

The other operations of the sequential search are constant, and we have also an extra $O(n)$ cost to compute the accumulated frequencies. Hence, assuming a previously sorted vocabulary, the overall cost of finding s and c is $O(n + \frac{n}{2^b} + 2^b \log_2 n)$. Therefore, the overall process has O(n) cost if $2^b \log_2 n = O(n)$. This holds provided $b \leq \log_2 n - \log_2 \log_2 n$, which is a reasonable condition[1].

Binary search

The binary search algorithm, using the *computeSizeS()* function, computes the size of the compressed text for two consecutive values of s in the middle of the interval that is checked in each iteration. Initially these two points are: $\lfloor 2^{b-1} \rfloor - 1$ and $\lfloor 2^{b-1} \rfloor$. Then the algorithm (using the heuristic of the existence of a unique minimum) can lead the search to the point that reaches the best compression ratio. In each new iteration, the search space is reduced by half and a new computation of the compression that is obtained with the two central points of the new interval is performed. Finally, the s and c values that minimize the length are returned.

```
BinaryFindBestS (b, acc)
(1)  //inputs: b value (2^b = c + s) and acc, the vector of accumulated frequencies
(2)  //output: The best s and c values
(3)  Lp ← 1;              //Lp and Up the lower and upper
(4)  Up ← 2^b − 1;        //points of the interval being checked
(5)  while Lp + 1 < Up
(6)       M ← ⌊(Lp+Up)/2⌋;
(7)       sizePp ← computeSizeS(M − 1, 2^b − (M − 1), acc); //size with M − 1
(8)       sizeM ← computeSizeS(M, 2^b − M, acc);          //size with M
(9)       if sizePp < sizeM then
(10)           Up ← M − 1;
(11)      else Lp ← M;
(12) if Lp < Up then    //Lp = Up − 1 and M = Lp
(13)      sizeNp ← computeSizeS(Up, 2^b − Up, acc); //size with M + 1
(14)      if sizeM < sizeNp then
(15)           bestS ← M;
(16)      else bestS ← Up;
(17) else bestS ← Lp;    //Lp = Up = M − 1
(18) bestC ← 2^b − bestS;
(19) return bestS, bestC;
```

[1]Given $b = 8$ and $n > 2^{12}$, it holds that: $\log_2 2^{12} - \log_2 \log_2 2^{12} = 8.42 \geq 8$

The most expensive possible sequence of calls to the *computeSizeS* algorithm occurs if the optimal s and c values are $s = 255$ and $c = 1$. In that case, *computeSizeS()* is called for the sequence of values $c = 2^{b-1}$, $c = 2^{b-2}$, $c = 2^{b-3}$,... , $c = 1$. Therefore, in the worst case, the cost of the *BinaryFindBestS()* algorithm is:

$$\frac{n}{2^b} + \sum_{i=1}^{b-1} \log_{2^{b-i}} n \;=\; \frac{n}{2^b} + \log_2 n \sum_{i=1}^{b-1} \frac{1}{b-i} \;=\; O\left(\frac{n}{2^b} + \log n \log b\right)$$

The other operations of the binary search are constant, and we have also an extra $O(n)$ cost to compute the accumulated frequencies. Hence the overall cost of finding s and c is $O(n + \log(n) \log(b))$. Since the maximum b of interest is such that $b = \lceil \log_2 n \rceil$ (because at this point we can code each symbol using a single stopper), the cost of optimization algorithm is at most $O(n + \log(n) \log\log(n)) = O(n)$, assuming that the vocabulary is already sorted. Therefore, the cost of computing the optimal s and c values is totally negligible, and computing the accumulated frequencies becomes the most time-consuming operation.

6.4 Encoding and decoding algorithms

Once the optimal s and c values for a given vocabulary are known, it is feasible to perform code generation. This encoding is usually done in a sequential fashion as shown in Table 6.1. However, an on-the-fly encoding process is also available as it happened in the case of End-Tagged Dense Code. Given a word rank i, its ℓ-byte codeword, can be easily computed in $O(\ell) = O(\log i)$ time.

Again, there is no need to store the codewords (in any form such as a tree) nor the frequencies in the compressed file. It is enough to store the plain words sorted by frequency and the value of s used in the compression process. Therefore, the vocabulary will be slightly smaller than in the case of a Huffman code, where some information about the shape of the tree must be stored (even when a canonical Huffman tree is used).

6.4.1 Encoding algorithm

The following pseudo-code presents the on-the-fly encoding algorithm. The algorithm outputs the bytes of each codeword one at a time from right to left. That is, it begins outputting the least significative bytes first.

```
Encode (i)
(1)  //input: i, the rank of the word in the vocabulary
(2)  //output: the codeword C_i from right to left
(3)  output i mod s;
(4)  x ← i div s;
(5)  while x > 0
(6)       x ← x − 1;
(7)       output(x mod c) + s;
(8)       x ← x div c;
```

A more efficient implementation of the on-the-fly encode algorithm that computes a codeword and returns also the codeword length k is also given next. This algorithm is the one we used when an on-the-fly encoding algorithm is needed (see Section 11.2).

```
Encode (i)
(1)  //input: i, the rank of the word in the vocabulary
(2)  //output: the codeword C_i from right to left, and its length k
(3)  k ← 1;
(4)  if i ≥ s
(5)       output i mod s;
(6)       x ← (i div s) − 1;
(7)       while x ≥ c
(8)            output (x mod c) + s;
(9)            x ← (x div c) − 1;
(10)           k ← k + 1;
(11)      output x + s;
(12)      k ← k + 1;
(13) else output i;
(14) return k;
```

6.4.2 Decoding algorithm

The first step of decompression is to load the words that compose the vocabulary to a vector. Since these words were saved ordered by frequency along with the s value and the compressed text during compression, the vocabulary of the decompressor is recovered already sorted by frequency. Once the sorted vocabulary is loaded, the decoding of codewords can begin.

In order to obtain the word w_i that corresponds to a given codeword c_i, the decoder can run a simple computation to obtain, from the codeword, the rank of the word i. Then, using the value i, it obtains the word from the vocabulary sorted

by frequency. A code c_i of ℓ bytes can be decoded in $O(\ell) = O(\log_c i)$ time[2] as follows:

The decoder uses a *base* table. This table indicates the rank of the first word of the vocabulary that is encoded with k bytes ($k \geq 1$). Therefore $base[1] = 0$, $base[2] = s$, $base[3] = s + sc$,... $base[k] = base[k-1] + sc^{k-2}$.

The *decode* algorithm receives a codeword x, and iterates over each byte of x. The end of the codeword can be easily recognized because its value is smaller than s. After the iteration, the value i holds the relative position of the word w_i among all the words of k bytes. Then the *base* table is used, and the final value is $i \leftarrow i + base[k]$. As a result, a position i is returned, and the decoded word w_i is obtained from $vocabulary[i]$.

```
Decode (base, x)
(1)  //input: x, the codeword to be decoded and the base table
(2)  //output: i, the position of the decoded word in the ranked vocabulary
(3)  i ← 0;
(4)  k ← 1; // number of bytes of the codeword
(5)  while x[k] ≥ s
(6)        i ← i × c + (x[k] − s);
(7)        k ← k + 1;
(8)  i ← i × s + x[k];
(9)  i ← i + base[k];
(10) return i;
```

6.5 Searching (s, c)-Dense Code

Searches over (s, c)-Dense Code are performed as in the case of End-Tagged Dense Code. As it was explained in Section 5.4, the search pattern is encoded and then searched through the compressed text using a Boyer-Moore type algorithm. A valid match is detected if the compressed pattern is found in the compressed text and it is preceded by a *stopper*.

The only difference between both techniques is the definition of the *stopper* and the *continuer* concepts. In End-Tagged Dense Code *continuers* and *stoppers* were defined as byte values in the ranges $[0, 2^{b-1} - 1]$ and $[2^{b-1}, 2^b - 1]$ respectively, whereas in (s, c)-Dense Code *stoppers* belong to the range $[0, s - 1]$ and *continuers* are byte values in the range $[s, 2^b - 1]$.

[2]Decoding takes $O(\log_c i)$ time if $c > 1$. If $c = 1$, it takes $O(i/2^b)$

CORPUS	Original Size	PH	(s,c)-DC	ETDC	TH	$DIFF_1$	$DIFF_2$
CALGARY	2,131,045	34.76	(197,59) 35.13	35.92	38.91	0.37	0.79
FT91	14,749,355	30.26	(193,63) 30.50	31.15	33.58	0.24	0.65
CR	51,085,545	29.21	(195,61) 29.45	30.10	32.43	0.24	0.65
FT92	175,449,235	30.49	(193,63) 30.71	31.31	34.08	0.22	0.60
ZIFF	185,220,215	31.83	(198,58) 32.02	32.72	35.33	0.19	0.70
FT93	197,586,294	30.61	(195,61) 30.81	31.49	34.30	0.20	0.68
FT94	203,783,923	30.57	(195,61) 30.77	31.46	34.28	0.20	0.69
AP	250,714,271	31.32	(189,67) 31.59	32.14	34.72	0.27	0.55
ALL_FT	591,568,807	30.73	(196,60) 30.88	31.56	34.16	0.15	0.68
ALL	1,080,719,883	32.05	(188,68) 32.24	32.88	35.60	0.19	0.64

(*)$DIFF_1$ shows the value (s, c)-DC - PH.
(*)$DIFF_2$ shows the value ETDC - (s, c)-DC.

Table 6.5: Comparison of compression ratio.

6.6 Empirical results

All the text collections in the experimental framework described in Section 2.7 were used. We compressed them applying Plain Huffman (PH), *(s,c)*-Dense Code ((s, c)-DC), End-Tagged Dense Code (ETDC) and Tagged Huffman (TH), using bytes ($b = 8$) as the target alfabet.

In Section 6.6.1, we compare the compression ratio achieved by each technique. In Section 6.6.2, we focus on time performance, comparing encoding time and compression time among the four techniques. Finally, in Section 6.6.3 we also compare the decompression speed obtained by (s, c)-Dense Code, End-Tagged Dense Code, Plain Huffman, and Tagged Huffman.

6.6.1 Compression ratio

Table 6.5 shows the compression ratio obtained by the different codes. We excluded the size of the compressed vocabulary in the results (this size is negligible and similar in all cases, although a bit smaller in (s, c)-DC and ETDC because only the sorted words are needed).

The second column of the table contains the original size of the processed corpus, the following four columns give the compression ratio for each method, the seventh column shows the small loss of compression of (s, c)-DC with respect to Plain Huffman, and the last column shows the difference between (s, c)-DC and End-Tagged Dense Code. The fourth column, which refers to (s, c)-DC, also gives the optimal (s, c) values.

As it can be seen in Table 6.5, Plain Huffman gets the best compression ratio

(as expected since it is an optimal prefix code). End-Tagged Dense Code always obtains better results than Tagged Huffman, with an improvement of up to 2.5 percentage points, as it was shown in Section 5.5.1. (s, c)-DC improves ETDC compression ratio by around 0.6 percentage points (as expected, since ETDC is a $(128, 128)$-DC), and it is worse than the optimal Plain Huffman only by less than 0.3 percentage points on average.

6.6.2 Encoding and compression times

As shown in the previous section, (s, c)-DC compression ratios are very close to the Plain Huffman ones. In this section we compare the (s, c)-DC and Plain Huffman encoding phases and measure code generation time and compression time. We also include End-Tagged Dense Code in the comparison because it is even simpler to build than (s, c)-Dense Code. In the case of Tagged Huffman, it works exactly as Plain Huffman, with the only difference of generating 2^{b-1}-ary trees instead of 2^b-ary trees as Plain Huffman does. This provokes a negligible loss of encoding speed, but worsens compression speed noticeably.

The model used for compressing a corpus in our experiments is described in Figure 6.4. It consists of three main phases.

1. The first phase is *vocabulary extraction*. The corpus is processed once in order to obtain all distinct words in it (n) and their number of occurrences. The result is a list of pairs $(word, frequency)$, which is then sorted by $frequency$. This phase is identical for Plain Huffman, (s, c)-Dense Code, and End-Tagged Dense Code.

2. In the second phase (*encoding*) each word in the vocabulary is assigned a codeword that minimizes average codeword length. This process is different for each method:

   - The Plain Huffman encoding phase is split into two main parts: *Creating the Huffman tree* uses the Huffman algorithm to build a tree where each leaf corresponds to one of the n words in the vocabulary, and the number of internal nodes is at most $\lceil \frac{n}{2^b-1} \rceil$. Then, starting from the root of the tree, the depth of each leaf is computed. Further details of this first part can be found in [MK95, MT96]. *Code assignment* starts at the bottom of the tree (longest codewords) and goes through all leaf nodes. Nodes in the same level are given codewords sequentially, and a jump of level is determined by using the previously computed leaf depths. During

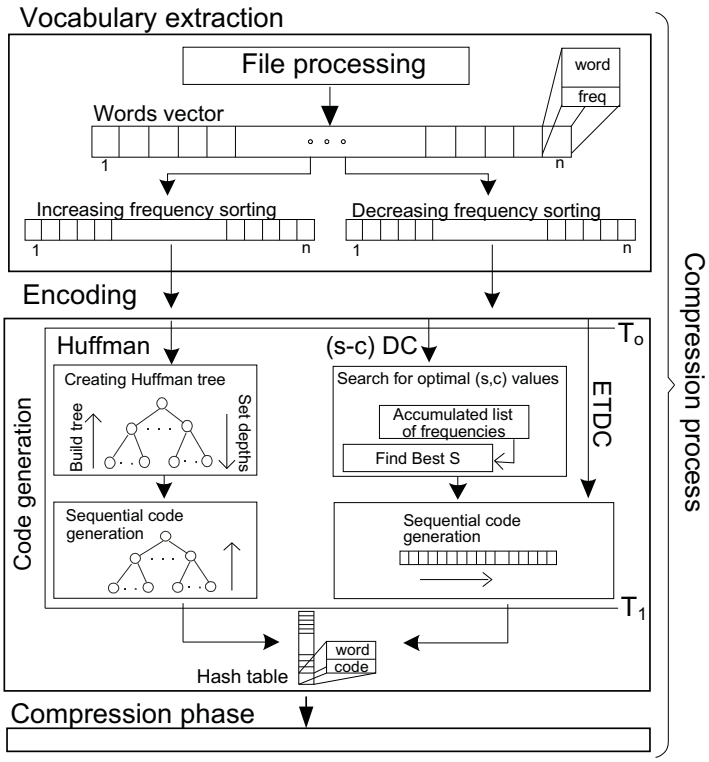


Figure 6.4: Vocabulary extraction and encoding phases.

this process, two vectors *base* and $first$, needed for fast decompression, are also initialized: $base[l] = x$ if x is the first node of the l^{th} level, and $first[l] = y$ if y is the first codeword of l bytes. Encoding takes $O([n + n/2^b] + [n/2^b] + [n/2^b + n])$=$O(n)$ time overall.

The same analysis can be applied to Tagged Huffman, in that case, encoding cost is $O([n + n/2^{b-1}] + [n/2^{b-1}] + [n/2^{b-1} + n])$=$O(n)$ time overall.

- The (s, c)-DC encoding phase has also two parts: The first computes the list of accumulated frequencies and searches for the optimal s and c values. Its cost is $O(n + \log(n) \log\log(n)) = O(n)$. After getting the optimal s and c values, (s, c) *sequential encoding* is performed. The overall cost is $O(n)$.

  Therefore, both Plain Huffman and (s, c)-Dense Code, run in linear time once the vocabulary is sorted. However, Huffman's constant is in practice larger because it involves more operations than just adding up

CORPUS	#words	n	ETDC (msec)	(s.c)-DC (msec)	PH (msec)	DIFF_1 (%)	DIFF_2 (%)
CALGARY	528,611	30,995	4.233	6.150	11.133	**31.165**	**44.760**
FT91	3,135,383	75,681	11.977	15.350	26.500	**21.976**	**42.075**
CR	10,230,907	117,713	21.053	25.750	49.833	**18.239**	**48.328**
FT92	36,803,204	291,427	52.397	69.000	129.817	**24.063**	**46.848**
ZIFF	40,866,492	284,892	44.373	56.650	105.900	**21.671**	**46.506**
FT93	42,063,804	295,018	52.813	69.725	133.350	**24.255**	**47.713**
FT94	43,335,126	269,141	52.980	71.600	134.367	**26.006**	**46.713**
AP	53,349,620	237,622	50.073	64.700	121.900	**22.607**	**46.924**
ALL_FT	124,971,944	577,352	103.727	142.875	260.800	**27.400**	**45.217**
ALL	229,596,845	886,190	165.417	216.225	402.875	**23.498**	**46.330**

(*)DIFF_1 shows the gain of ETDC over (s, c)-DC.
(*)DIFF_2 shows the gain of (s, c)-DC over PH.

Table 6.6: Code generation time comparison.

frequencies.

- The encoding phase is even simpler in ETDC than in (s, c)-Dense Code, because ETDC does not have to search for the optimal s and c values (they are fixed to 128). Therefore only the *sequential code generation* phase is performed. It costs $O(n)$ time overall.

In all cases, the result of the encoding section is a *hash table* of pairs $(word, codeword)$.

3. The third phase, *compression*, processes again the whole source text. For each input word, the compression process looks for it inside the *hash table* and outputs its corresponding codeword.

Given that the vocabulary extraction phase, the process of building the hash table of pairs, and the compression phase are present in Plain Huffman, ETDC and (s, c)-DC, we first measured only code generation time ($T_1 - T_0$ in Figure 6.4), to compare the three techniques. Lastly, we also measure the time needed to complete the whole compression process.

Encoding time comparison

Table 6.6 shows the results obtained regarding to code generation time. The first column indicates the corpus processed, the second the number of words in the corpus, and the third the number of distinct words in the vocabulary. The fourth, fifth, and sixth columns give the encoding time (in milliseconds) for ETDC, (s, c)-DC, and Plain Huffman respectively. The seventh column presents the gain (in percentage) of ETDC over (s, c)-DC. Finally, the last column shows the gain (in percentage) of (s, c)-DC over Plain Huffman.

ETDC takes advantage of its simpler encoding phase with respect to (s, c)-DC,

and reduces its encoding time by about 25%. This difference corresponds exactly to the amount of time needed to compute the optimal s and c values, and corresponds mainly to the process of computing the vector of accumulated frequencies. With respect to Plain Huffman, ETDC decreases the encoding time by about 60%.

(s,c)-DC code generation process is always about 45% faster than Plain Huffman. Although the encoding is in both methods $O(n)$ under reasonable conditions, (s,c)-DC performs simpler operations. Computing the list of accumulated frequencies and searching for the best (s,c) pair only involve elemental operations, while the process of building a canonical Huffman tree has to deal with the tree structure.

Compression time comparison

As it happened in the previous chapter (where we compared End-Tagged Dense Code with Plain Huffman), the advantage obtained in encoding time with respect to Plain Huffman, is compensated by the better compression ratio of Plain Huffman. As a result, differences in compression speed among our "dense" techniques and the Huffman-based methods are minimal as it is shown in Table 6.7.

CORPUS	Compr. time (sec)				Compr. Speed (Kbytes/sec)			
	PH	(s,c)-DC	ETDC	TH	PH	(s,c)-DC	ETDC	TH
CALGARY	0.415	0.405	0.393	0.415	5,135.05	5,261.84	5,417.91	5,135.05
FT91	2.500	2.493	2.482	2.503	5,899.74	5,915.52	5,943.33	5,891.89
CR	7.990	7.956	7.988	8.040	6,393.69	6,420.81	6,395.46	6,353.92
FT92	29.243	29.339	29.230	29.436	5,999.80	5,980.12	6,002.37	5,960.45
ZIFF	30.354	30.620	30.368	30.725	6,101.95	6,048.99	6,099.29	6,028.32
FT93	32.915	33.031	32.783	33.203	6,002.93	5,981.77	6,027.12	5,950.80
FT94	33.874	33.717	33.763	33.700	6,015.98	6,044.01	6,035.81	6,047.00
AP	42.641	42.676	42.357	42.663	5,879.62	5,874.83	5,919.06	5,876.57
ALL_FT	99.889	100.570	100.469	101.471	5,922.28	5,882.16	5,888.10	5,829.92
ALL	191.396	191.809	191.763	192.353	5,646.53	5,634.36	5,635.70	5,618.42

a) compression time and compression speed.

CORPUS	(s,c)-DC - PH (%)	(s,c)-DC - ETDC (%)	(s,c)-DC - TH (%)
CALGARY	2.469	-2.881	2.469
FT91	0.267	-0.468	0.401
CR	0.424	0.396	1.053
FT92	-0.328	-0.371	0.330
ZIFF	-0.868	-0.825	0.343
FT93	-0.352	-0.753	0.520
FT94	0.466	0.136	0.010
AP	-0.081	-0.747	0.033
ALL_FT	-0.677	-0.101	0.896
ALL	-0.215	-0.024	0.284

b) Differences with respect to (s,c)-DC.

Table 6.7: Compression speed comparison.

Table 6.7.a) gives results for each technique with respect to compression time and compression speed. The first column in that table indicates the corpus processed. The following four columns show compression time (in seconds) and the last four

columns show the compression speed (in Kbytes per second) for Plain Huffman, (s, c)-DC, ETDC, and Tagged Huffman respectively.

Table 6.7.b) compares ETDC, Plain Huffman, and Tagged Huffman against (s, c)-DC. Columns from two to four show the gain (in percentage) in compression speed of (s, c)-DC over the other techniques. Positive values mean that (s, c)-DC is better than the others. Although differences among the four techniques are very small, it can be seen that End-Tagged Dense Code is the fastest technique, and that Plain Huffman is slightly faster in compression than (s, c)-DC. It is also shown that Tagged Huffman is beaten by its competitors in all corpora.

It is interesting to note that, even having a worse compression ratio than (s, c)-DC, ETDC is faster. This advantage of ETDC arises because it can use fast bitwise operations, whereas (s, c)-DC needs to use slower division and modulus operations.

6.6.3 Decompression time

The decompression process is almost identical for the Huffman-based techniques and (s, c)-DC (ETDC is decompressed assuming that it is a $(128, 128)$-DC). The process starts by loading the words of the vocabulary into a vector V. For decoding a codeword, (s, c)-DC also needs the value of s used in compression, whereas Plain Huffman and Tagged Huffman need to load the vectors $base$ and $first$. Next, the compressed text is read and each codeword is replaced by its corresponding uncompressed word. Since it is possible to detect the end of a codeword by using either the s value (in (s, c)-DC) or the $first$ vector (in Plain and Tagged Huffman), decompression is performed codeword-wise. Given a codeword C, a simple decoding algorithm obtains the position i of the word in the vocabulary, such that $V[i]$ is the uncompressed word that corresponds to codeword C. Decompression takes $O(v)$ time, being v the size (in bytes) of the compressed text.

All the corpora in our experimental framework were decompressed using Plain Huffman, (s, c)-DC, ETDC, and Tagged Huffman. Results in Table 6.8.a) compare decompression time and decompression speed for the four techniques. The first column shows the corpus being decompressed. Next four columns present decompression time (in seconds) for each method, and columns from the sixth to the ninth show decompression speed (in Kbytes per second).

Table 6.8.b) compares ETDC, Plain Huffman, and Tagged Huffman against (s, c)-DC. The first column shows the corpus being decompressed and the remainder three columns in that table show the gain (in percentage) in decompression speed of (s, c)-DC over Plain Huffman, End-Tagged Dense Code, and Tagged Huffman

CORPUS	Decompr. time (sec)				Decompr. Speed (Kbytes/sec)			
	PH	(s,c)-DC	ETDC	TH	PH	(s,c)-DC	ETDC	TH
CALGARY	0.088	0.097	0.085	0.092	24,125.04	22,045.29	25,071.12	23,247.76
FT91	0.577	0.603	0.570	0.575	25,576.92	24,446.44	25,876.06	25,651.05
CR	1.903	1.971	1.926	1.968	26,851.80	25,917.13	26,530.29	25,964.70
FT92	7.773	7.592	7.561	7.801	22,573.08	23,109.75	23,204.16	22,489.89
ZIFF	8.263	7.988	7.953	8.081	22,414.71	23,187.95	23,289.77	22,919.24
FT93	8.406	8.437	8.694	8.657	23,506.19	23,418.63	22,727.40	22,824.75
FT94	8.636	8.690	8.463	8.825	23,596.34	23,450.39	24,080.82	23,091.66
AP	11.040	11.404	11.233	11.637	22,709.63	21,985.25	22,318.78	21,544.31
ALL_FT	24.798	25.118	24.500	26.280	23,855.99	23,552.06	24,145.67	22,510.23
ALL	45.699	46.698	46.352	47.156	23,648.88	23,142.74	23,315.50	22,918.11

a) compression time and compression speed.

CORPUS	(s,c)-DC - PH (%)	(s,c)-DC - ETDC (%)	(s,c)-DC - TH (%)
CALGARY	-8.621	-12.069	-5.172
FT91	-4.420	-5.525	-4.696
CR	-3.481	-2.311	-0.183
FT92	2.378	-0.407	2.756
ZIFF	3.450	-0.437	1.172
FT93	-0.373	3.041	2.602
FT94	-0.619	-2.618	1.554
AP	-3.190	-1.494	2.047
ALL_FT	-1.274	-2.458	4.628
ALL	-2.140	-0.741	0.980

b) Differences with respect to (s,c)-DC.

Table 6.8: Decompression speed comparison.

respectively. It can be seen that Both End-Tagged Dense Code and Plain Huffman are a bit faster in decompression than (s,c)-Dense Code, whereas Tagged Huffman is clearly the slower technique in decompression.

6.6.4 Search time

Empirical results shown in Section 5.5.4 give End-Tagged Dense Code a little advantage $(2-7\%)$ in search speed with respect to Tagged Huffman, when fixed-length codewords are searched.

The algorithm to search in (s,c)-Dense Code is exactly the same as in ETDC. The same good results can be expected, but improved from the fact that (s,c)-Dense Code obtains better compression than ETDC, and therefore the text to be scanned is shorter.

We present in Table 6.9 the results of searching for some single word patterns in the ALL compressed corpus. The first three columns give the length of the codeword associated to the word that is being searched, the word itself and the number of occurrences of that word in the text. Columns four, five, and six show the search time (in seconds) for the three techniques. The last two columns give the decrease of search time (in percentage) of (s,c)-Dense Code with respect to End-Tagged Dense Code and Tagged Huffman respectively.

The words searched in the experiments are the most and least frequent words that are encoded with 1, 2, or 3 bytes with the three compression techniques.

code length	word	occur.	(s,c)-DC (sec)	ETDC (sec)	TH (sec)	DIFF$_1$ (%)	DIFF$_2$ (%)
1	the	8,205,778	5.308	5.452	5.759	2.636	7.826
1	were	356,144	4.701	5.048	5.182	6.856	9.273
2	sales	88,442	2.526	2.679	2.800	5.711	9.779
2	predecessor	2,775	2.520	2.666	2.736	5.476	7.895
3	resilience	612	1.671	1.779	1.858	6.061	10.059
3	behooves	22	1.613	1.667	1.701	3.245	5.189

(*)DIFF$_1$ and DIFF$_2$ show the gain of (s,c)-DC with respect to ETDC and TH respectively.

Table 6.9: Searching time comparison.

It can be seen that, due to the better compression ratio, (s,c)-Dense Code overcomes End-Tagged Dense Code in searches, achieving a decrease of search time up to 6%. Therefore, it also overcomes Tagged Huffman search speed in about $5-10$ percentage points.

Results for random searches are given in Table 6.10. $10,000$ random words, appearing at least twice in a corpus, were randomly taken from each corpus and their corresponding codeword was searched inside the compressed corpus. The second and third columns in that table, show the average-time ($\overline{time}$) and the standard deviation (σ) respectively for (s,c)-Dense Code. Average-time and standard deviation for End-Tagged Dense Code and Tagged Huffman are shown in columns 4 to 7. Finally, the last two columns show the difference (in percentage) between (s,c)-Dense Code and Tagged Huffman, and between (s,c)-Dense Code and End-Tagged Dense Code respectively.

Tagged Huffman improves its search results when searching for random single-word patterns in large corpus. As we have already explained in Section 5.5.4, Tagged Huffman uses longer codewords than End-Tagged Dense Code and (s,c)-Dense Code. This gives it a small advantage when those codewords are searched using a Boyer-Moore type searching algorithm. However, note that (s,c)-Dense Code obtains better results than Tagged Huffman in general, with an improvement around 7 percentage points. In practice, Tagged Huffman only improves (s,c)-Dense Code in the large FT_ALL and ALL collections, where it takes advantage of using many long codewords.

With respect to End-Tagged Dense Code, (s,c)-Dense Code obtains an advantage of about $2-3$ percentage points in all corpus.

CORPUS	(s,c)-DC		ETDC		TH		DIFF (%)	
	$\overline{\text{time}}$	σ	$\overline{\text{time}}$	σ	$\overline{\text{time}}$	σ	TH - (s,c)-DC	ETDC - (s,c)-DC
FT91	0.023	0.006	0.024	0.006	0.024	0.005	4.348	4.348
CR	0.072	0.015	0.073	0.016	0.077	0.012	6.944	1.389
FT92	0.250	0.038	0.257	0.044	0.267	0.046	6.800	2.800
ZIFF	0.275	0.041	0.283	0.047	0.292	0.052	6.182	2.909
FT93	0.283	0.049	0.291	0.045	0.299	0.052	5.654	2.827
FT94	0.291	0.039	0.300	0.047	0.306	0.059	5.155	3.093
AP	0.376	0.056	0.382	0.066	0.380	0.048	1.064	1.596
ALL_FT	0.844	0.091	0.867	0.101	0.760	0.141	-9.953	2.725
ALL	1.610	0.176	1.650	0.195	1.390	0.250	-13.665	2.484

Table 6.10: Searching for random patterns: time comparison.

6.7 Summary

(s,c)-Dense Code, a simple method for compressing natural language Text Databases with several advantages over the existing techniques was presented. This technique is a generalization of the previously presented End-Tagged Dense Code, and improves its compression ratio by adjusting the parameters s and c to the distribution of frequencies of the text to be compressed.

Some empirical results comparing (s,c)-DC with Huffman codes are shown. In compression ratio, our new code is strictly better than Tagged Huffman Code (by 3.5 percentage points in practice), and it is only 0.3 percentage points worse than the optimal Plain Huffman Code. The new code is simpler to build than Huffman Codes and can be built in around half the time. In compression and decompression speed, (s,c)-DC is slightly slower than End-Tagged Dense Code and Plain Huffman, however differences are minimal. This makes (s,c)-DC a real alternative to Plain Huffman in situations where a simple, fast and good compression method is required. Moreover, as in Tagged Huffman, (s,c)-DC enables fast direct searches on the compressed text, which improves Plain Huffman searching efficiency and capabilities. However, searches in Tagged Huffman are around 5-10% slower than in (s,c)-DC. Tagged Huffman is only faster than (s,c)-Dense Code when we perform random searches inside very large texts (the random words are encoded with longer codewords in Tagged Huffman).

Figure 6.5 summarizes compression ratio, encoding time, compression and decompression time, and search time for the different methods. In that figure, the measures obtained in the AP corpus are shown normalized to the worst value. Moreover, since we show compression ratio and time comparisons, the lower the value in a bar, the better the compression techniques.

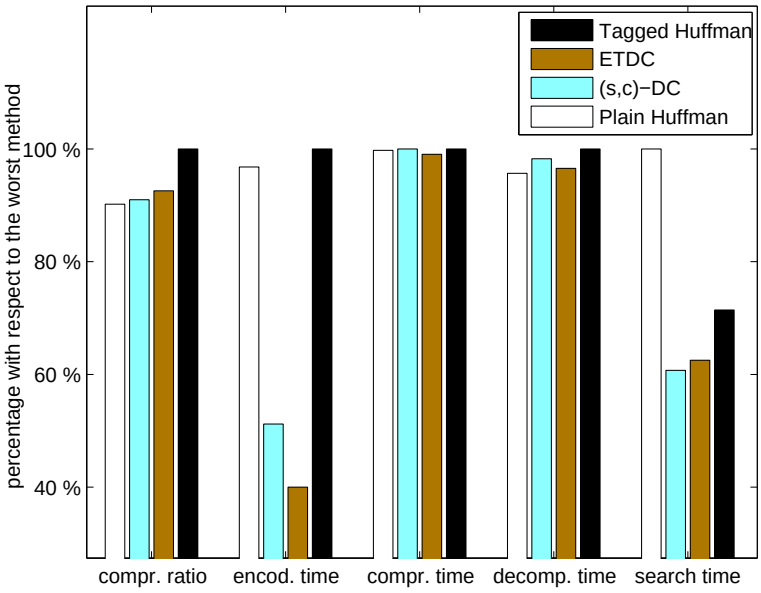


Figure 6.5: Comparison of "dense" and Huffman-based codes.

Note that Plain Huffman remains interesting because it has the better compression ratio. However, Tagged Huffman has been overcome by both End-Tagged Dense Code and (s, c)-Dense Code in all concerns: compression ratio, encoding speed, compression and decompression speed, and search speed.

7

New bounds on D-ary Huffman coding

7.1 Motivation

Bounding the compression that a D-ary Huffman code achieves is an interesting problem. In this chapter, we provide new analytical bounds that can be obtained by using End-Tagged Dense Code and (s, c)-Dense Code. The new bounds permit us to approximate the compression achieved by a D-ary Huffman code without having to run Huffman's algorithm.

Section 7.2 and Section 7.3 explain how the average codeword length of End-Tagged Dense code and (s, c)-Dense code can be used to bound Huffman. In Section 7.4, we describe how the *zero*-order entropy (H_0^D) permits bounding Huffman compression.

An interesting contribution of (s, c)-Dense Code is that analytical bounds for a D-ary Huffman code can be obtained for different distributions of symbol frequencies. In Section 7.5, an interesting distribution, called Zipf-Mandelbrot's distribution, is assumed and analytical bounds to D-ary Huffman are obtained. Finally, the new bounds obtained are applied to real and theoretical text collections in Sections 7.6 and 7.7.

7.2 Using End-Tagged Dense Code to bound Huffman Compression

An interesting property of End-Tagged Dense Code is that it can be used as a bound for the compression that can be obtained with a Huffman code. As shown in Section 5.2, End-Tagged Dense Code uses all the possible combinations of all bits, except for the first bit, that is used as a flag as in the Tagged Huffman Code. Therefore, calling E_b the average codeword length of the End-Tagged Dense Code that uses symbols of b bits, and defining $L_h(2^b)$ and T_b respectively as the average codeword length of Plain Huffman and Tagged Huffman (with symbols of b bits) we have:

$$E_{b+1} \leq L_h(2^b) \leq E_b \leq T_b \leq E_{b-1}$$

It is easy to see that $E_{b+1} \leq L_h(2^b)$ since End-Tagged Dense Code, with $b+1$ bits, uses all the 2^b combinations of $b+1-1$ bits, while in Plain Huffman some of the 2^b combinations of b bits are not used. The same reasoning can be applied to see that $E_b \leq T_b$.

On the other hand, E_b is a prefix code of b bits and Huffman is the optimal prefix code using b bits, thus it holds that $L_h(2^b) \leq E_b$. To understand that $T_b \leq E_{b-1}$ it is only needed to consider Tagged Huffman over b bits as a Huffman code over $b-1$ bits, and therefore it is always better than End-Tagged Dense Code over $b-1$ bits.

Figure 7.1 shows the shape of a Tagged Huffman tree. It is also shown how the shape of a tree representing the codewords generated by End-Tagged Dense Code would be. It can be seen that End-Tagged Dense Code achieves a smaller compressed text size than Tagged Huffman, since an internal node will have 256 children: 128 internal nodes and also 128 leaf nodes. However, in Tagged Huffman, an internal node can have less than 128 children, some of them (z) will be leaves and the others, at most ($128 - z$), will be internal nodes. Therefore End-Tagged Dense Code has, at least, the same number of codewords of a given length l than Tagged Huffman (where l corresponds to a level in the tree).

7.3 Bounding Plain Huffman with (s, c)-Dense Code

In this section, it is shown that the average codeword length of a D-ary Huffman code ($L_h(D)$) can be lower and upper bounded by the average codeword length

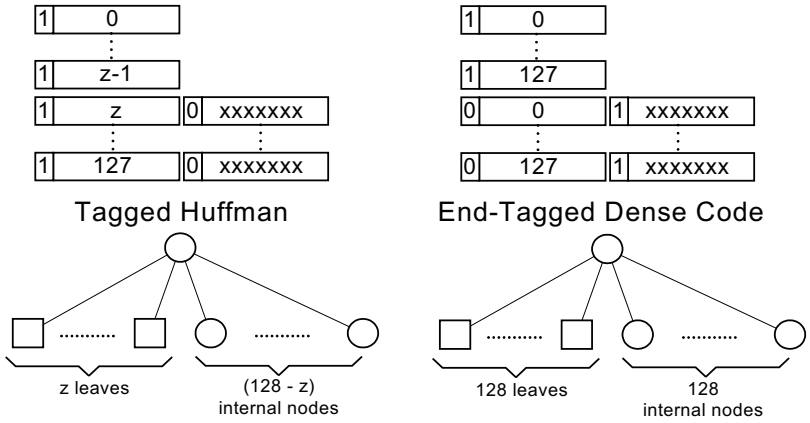


Figure 7.1: Comparison of Tagged Huffman and End-Tagged Dense Code.

($L_d(s,c)$) that is obtained by (s,c)-Dense Code. For the upper bound we use $L_d(s,c)$ for some values such that $D = s+c$, and for the lower bound we use $L_d(s,c)$, having $2D = s + c$.

As shown in Section 6.2, (s,c)-Dense Code is a prefix code, and it is well-known that Huffman [Huf52] is the optimal prefix code. Therefore, $L_d(s,c) \geq L_h(D)$. Moreover a D-ary Huffman code can be seen as a (D,D) stop-cont code provided that D is added to all digits in a codeword, except the last one. Therefore, $L_h(D)$ cannot be smaller than the average codeword length of a (D,D)-Dense Code. Therefore, $L_d(D,D) \leq L_h(D)$ clearly holds.

Summarizing, (s,c)-Dense Code provides new bounds to a D-ary Huffman code in the following way:

$$L_d(D,D) \;\leq\; L_h(D) \;\leq\; L_d(s,c) \;\text{ for any } D = s + c \tag{7.1}$$

7.4 Analytical entropy-based bounds

From the *Noiseless Coding Theorem* [SW49] it holds that $H_0^D \leq L_h(D) \leq H_0^D + 1$, where $D = 2$ and, as we showed in Section 2.2,

$$H_0^D = \; -\sum_{i=1}^{n} p(x_i) \;\log_D\; p(x_i)$$

is the D-ary zero-order entropy of the words in the vocabulary. Therefore H_0^D is a lower bound of $L_h(D)$.

If Zipf-Mandelbrot's law (see Section 2.4.2) is assumed, H_0^D can be computed as follows:

$$\begin{aligned} H_0^D &= \sum_{i\geq 1} p_i \log_D \frac{1}{p_i} = \sum_{i\geq 1} \frac{A}{(1+Ci)^\theta} \log_D \frac{1}{\frac{A}{(1+Ci)^\theta}} \\ &= A\sum_{i\geq 1}\left[\frac{-\log_D A}{(1+Ci)^\theta} + \frac{\log_D (1+Ci)^\theta}{(1+Ci)^\theta}\right] \\ &= -\log_D A \sum_{i\geq 1} \frac{A}{(1+Ci)^\theta} + \sum_{i\geq 1} \frac{A\theta \log_D (1+Ci)}{(1+Ci)^\theta} \\ &= -\log_D A \sum_{i\geq 1} p_i + A\theta \sum_{i\geq 1} \frac{\log_D (1+Ci)}{(1+Ci)^\theta} \\ &= -\log_D A + A\theta \sum_{i\geq 1} \frac{\log_D (1+Ci)}{(1+Ci)^\theta} \\ &= \frac{-\theta A \sum_{i\geq 1} \frac{-\ln(1+Ci)}{(1+Ci)^\theta} - \ln A}{\ln D} \end{aligned} \tag{7.2}$$

From Equation 2.2, it is known that: $A = \frac{1}{\sum_{i\geq 1} \frac{1}{(1+Ci)^\theta}} = \frac{1}{\zeta_C(\theta)}$. At this point we resort to derivation of $\zeta_C(\theta)$ with respect to θ

$$\zeta_C'(\theta) = \sum_{i\geq 1} \frac{-(1+Ci)^\theta \ln(1+Ci)}{(1+Ci)^{2\theta}} = \sum_{i\geq 1} \frac{-\ln(1+Ci)}{(1+Ci)^\theta}$$

And then we substitute in (7.2):

$$H_0^D = \frac{-\theta \frac{1}{\zeta_C(\theta)} \zeta_C'(\theta) - \ln \frac{1}{\zeta_C(\theta)}}{\ln D} = \frac{-\theta \frac{\zeta_C'(\theta)}{\zeta_C(\theta)} + \ln \zeta_C(\theta)}{\ln D}$$

Then a lower bound to $L_h(D)$ based on the entropy value is:

$$L_h(D) \geq \frac{-\theta \frac{\zeta_C'(\theta)}{\zeta_C(\theta)} + \ln \zeta_C(\theta)}{\ln D} \tag{7.3}$$

and an upper bound is obtained as:

$$L_h(D) \leq H_0^D + 1 = 1 + \frac{-\theta \frac{\zeta_C'(\theta)}{\zeta_C(\theta)} + \ln \zeta_C(\theta)}{\ln D} \tag{7.4}$$

7.5 Analytical bounds with (s,c)-Dense Code

From Equation 6.3 the average codeword length of (s,c)-Dense Code for a given distribution of word frequencies which follows Zipf-Mandelbrot's law is:

$$L_d(s,c) = 1 + \sum_{k\geq 1}\sum_{i\geq W_k^s+1} p_i = 1 + A\sum_{k\geq 1}\sum_{i\geq W_k^s+1}\frac{1}{(1+Ci)^\theta} \tag{7.5}$$

Note that $f(i) = \frac{1}{(1+Ci)^\theta}$, having $C > 0$, $\theta > 1$ and $i \geq 1$, is a continuous, positive and decreasing function. Therefore the series $\sum_{i\geq 1} f(i) = \sum_{i\geq 1}\frac{1}{(1+Ci)^\theta}$ converges if $\theta > 1$ since $\sum_{i\geq 1}\frac{1}{(1+Ci)^\theta} \leq \frac{1}{C^\theta}\sum_{i\geq 1}\frac{1}{i^\theta}$.

By applying the Integration Theorem, $\int_1^\infty f(x)dx$ exists, and it is possible to bound the series $\sum_{k=n+1}^\infty f_k$ as follows: $\int_{i+1}^\infty f(x)dx \leq \sum_{k\geq i+1} f_k \leq \int_i^\infty f(x)dx$.

As a result, from 7.5 it is obtained:

$$L_d(s,c) \leq 1 + A\sum_{k\geq 1}\int_{W_k^s}^\infty \frac{1}{(1+Cx)^\theta}dx \tag{7.6}$$

$$L_d(s,c) \geq 1 + A\sum_{k\geq 1}\int_{W_k^s+1}^\infty \frac{1}{(1+Cx)^\theta}dx \tag{7.7}$$

7.5.1 Upper bound

As shown in Equation 7.1, it is possible to achieve an upper bound to $L_h(D)$ using $L_d(s,c)$. We use Equation 7.6 and operate in the summation. Note that we assume $c > 1$. The case $c = 1$ is treated later.

$$\sum_{k\geq 1}\int_{W_k^s}^\infty \frac{1}{(1+Cx)^\theta}dx = \sum_{k\geq 1}\frac{(1+Cx)^{-\theta+1}}{C(-\theta+1)}\Big]_{W_k^s}^\infty = \sum_{k\geq 1}\frac{-1}{C(1-\theta)(1+CW_k^s)^{\theta-1}}$$

$$\begin{aligned}
&= \frac{1}{C(\theta-1)}\sum_{k\geq 1}\frac{1}{\left(1+Cs\frac{c^k-1}{c-1}\right)^{\theta-1}} \\
&= \frac{1}{C(\theta-1)}\frac{(c-1)^{\theta-1}}{s^{\theta-1}}\sum_{k\geq 1}\frac{1}{\left(\frac{(c-1)}{s}+C(c^k-1)\right)^{\theta-1}} \\
&< \frac{1}{C(\theta-1)}\frac{(c-1)^{\theta-1}}{s^{\theta-1}}\frac{1}{C^{\theta-1}}\sum_{k\geq 1}\frac{1}{(c^k-1)^{\theta-1}}
\end{aligned} \tag{7.8}$$

Equation 7.8 can be simplified, since:

$$\begin{aligned}
\sum_{k\geq 1}\frac{1}{(c^k-1)^{\theta-1}} &= \sum_{k\geq 1}\frac{1}{(c^k(1-\frac{1}{c^k}))^{\theta-1}} = \sum_{k\geq 1}\frac{1}{c^{k(\theta-1)}(1-\frac{1}{c^k})^{\theta-1}} \\
&< \sum_{k\geq 1}\frac{1}{c^{k(\theta-1)}(1-\frac{1}{c})^{\theta-1}} = \left(1-\frac{1}{c}\right)^{1-\theta}\sum_{k\geq 1}\frac{1}{c^{k(\theta-1)}} \\
&= \left(1-\frac{1}{c}\right)^{1-\theta}\frac{\frac{1}{c^{\theta-1}}}{1-\frac{1}{c^{\theta-1}}} = \left(1-\frac{1}{c}\right)^{1-\theta}\frac{c^{1-\theta}}{1-c^{1-\theta}} \\
&= \frac{(c-1)^{1-\theta}}{1-c^{1-\theta}} \qquad (7.9)
\end{aligned}$$

From Equations 7.6, 7.8, and 7.9 it is obtained that:

$$\begin{aligned}
L_d(s,c) &< 1 + A\frac{1}{C(\theta-1)}\frac{1}{s^{\theta-1}}\frac{1}{C^{\theta-1}}\frac{1}{1-c^{1-\theta}} \\
&= 1 + \frac{1}{C^{\theta}\ (\theta-1)\ \zeta_C(\theta)\ s^{\theta-1}\ (1-c^{1-\theta})} = \vartheta \qquad (7.10)
\end{aligned}$$

Such equation represents an upper bound for $L_d(s,c)$. Therefore in order to obtain a minimal upper bound we substitute $c = D-s$ and then we resort to differentiation in s.

$$\frac{\partial\vartheta}{\partial s} = \frac{-\left[s^{\theta-1}((1-\theta)(D-s)^{-\theta}) + (\theta-1)s^{\theta-2}(1-(D-s)^{1-\theta})\right]}{C^{\theta}\ (\theta-1)\ \zeta_C(\theta)\ \ s^{(\theta-1)2}\ [1-(D-s)^{1-\theta}]^2}$$

And now we solve for $\frac{\partial\vartheta}{\partial s} = 0$, that is:

$$\begin{aligned}
0 &= s^{\theta-1}(1-\theta)(D-s)^{-\theta} + (\theta-1)s^{\theta-2}(1-(D-s)^{1-\theta}) \\
&\quad \text{since, } 0<c<D \text{ and } \theta>1 \\
0 &= s^{\theta-2}(1-\theta)\left[s(D-s)^{-\theta} - \left(1-(D-s)^{(1-\theta)}\right)\right] \\
0 &= s(D-s)^{-\theta} - \left(1-(D-s)^{1-\theta}\right) \\
1 &= s(D-s)^{-\theta} + (D-s)(D-s)^{-\theta} \\
1 &= (D-s)^{-\theta}[(D-s)+s] \\
1 &= (D-s)^{-\theta}D \\
D-s &= D^{\frac{1}{\theta}} \\
s &= D - D^{\frac{1}{\theta}}, \text{ is the } s \text{ value that gives the best upper bound.}
\end{aligned}$$

Then we obtain that the optimal c value is: $c = D - s = D^{\frac{1}{\theta}}$.

Therefore, a minimal upper bound for $L_d(s, c)$ is obtained by substituting the optimal s and c values in Equation 7.10, that is:

$$\min_s L_d(s,c) \quad \leq \quad 1 + \frac{1}{C^\theta(\theta-1)\zeta_C(\theta)(D - D^{\frac{1}{\theta}})^{\theta-1}(1 - D^{\frac{1}{\theta}-1})} \tag{7.11}$$

If we assume $c = 1$ then $W_k^s = k(D-1)$, hence from Equation 7.6 we obtain:

$$\sum_{k\geq 1} \int_{W_k^s}^{\infty} \frac{1}{(1+Cx)^\theta} dx \quad = \quad \frac{1}{\theta-1}\frac{1}{C^\theta}\frac{1}{(D-1)^{\theta-1}} \sum_{k\geq 1} \frac{1}{k^{\theta-1}} \tag{7.12}$$

Since the summation above diverges if $1 < \theta \leq 2$, no upper bound is obtained for $c = 1$ and $1 < \theta \leq 2$. However, given $\theta > 2$,

$$\sum_{k\geq 1} \frac{1}{k^{\theta-1}} = 1 + \sum_{k\geq 2} \frac{1}{k^{\theta-1}} \leq 1 + \int_1^{\infty} k^{1-\theta} dk = \frac{\theta-1}{\theta-2} \tag{7.13}$$

Therefore, from Equations 7.6, 7.12, and 7.13 an upper bound is obtained as:

$$L_d(D-1,1) \quad \leq \quad 1 + \frac{1}{(\theta-2)\ \zeta_C(\theta)\ C^\theta\ (D-1)^{\theta-1}}, \quad \theta > 2 \tag{7.14}$$

7.5.2 Lower bound

Similarly to the way the upper bound was obtained, it is also possible to obtain a lower bound. Using Equation 7.7 we have:

$$L_d(s,c) \quad \geq \quad 1 + A \sum_{k\geq 1} \int_{W_k^s+1}^{\infty} \frac{1}{(1+Cx)^\theta} dx$$

For $c = 1$ the compression obtained by (s, c)-Dense Code is poor, so it is not possible to guarantee that it will obtain a lower bound to $L_h(D)$. However, assuming $c > 1$ we obtain:

$$\begin{aligned}
&\int_{W_k^s+1}^{\infty} \frac{1}{(1+Cx)^\theta} dx = \frac{(1+Cx)^{-\theta+1}}{C(-\theta+1)}\Bigg]_{W_k^s+1}^{\infty} = \\
&= \frac{1}{C(1-\theta)} \frac{-1}{[1+C(W_k^s+1)]^{\theta-1}} = \frac{1}{C(\theta-1)} \frac{1}{\left[1+C(s\frac{c^k-1}{c-1}+1)\right]^{\theta-1}} \\
&= \frac{(c-1)^{\theta-1}}{C(\theta-1)} \frac{1}{[(c-1)+C(s(c^k-1)+(c-1))]^{\theta-1}}
\end{aligned}$$

Therefore we obtain that:

$$L_d(s,c) \geq 1 + A\sum_{k\geq 1} \frac{(c-1)^{\theta-1}}{C(\theta-1)} \frac{1}{\left[(c-1)+C(s(c^k-1)+(c-1))\right]^{\theta-1}} \tag{7.15}$$

At this point, note that given $c > 1$, $s \geq 1$ and $k \geq 1$

$$\begin{aligned}
&(c-1)+C(s(c^k-1)+(c-1)) = (c-1)(1+C)+Cs(c^k-1) \leq \\
&\leq (c^k-1)(1+C)+Cs(c^k-1) = (c^k-1)(1+C+Cs) \text{ then} \\
&\left[(c-1)+C(s(c^k-1)+(c-1)\right]^{\theta-1} \leq \left[(c^k-1)(1+C+Cs)\right]^{\theta-1}
\end{aligned}$$

and

$$\frac{1}{[(c-1)+C[s(c^k-1)+(c-1)]]^{\theta-1}} \quad \geq \quad \frac{1}{[(c^k-1)(1+C+Cs)]^{\theta-1}}$$

As a result, Equation 7.15 can be transformed into:

$$\begin{aligned}
L_d(s,c) &\geq 1 + A\sum_{k\geq 1} \frac{(c-1)^{\theta-1}}{C(\theta-1)} \frac{1}{\left[(1+C+Cs)(c^k-1)\right]^{\theta-1}} \\
&\geq 1 + A\frac{(c-1)^{\theta-1}}{C(\theta-1)} \frac{1}{(1+C+Cs)^{\theta-1}} \sum_{k\geq 1} \frac{1}{(c^k)^{\theta-1}} \\
&= 1 + \frac{(c-1)^{\theta-1}}{\zeta_C(\theta)\ C(\theta-1)\ (1+C+Cs)^{\theta-1}} \frac{c^{1-\theta}}{1-c^{1-\theta}} \\
&= 1 + \frac{1}{\zeta_C(\theta)\ C(\theta-1)\ (1+C+Cs)^{\theta-1}} \frac{(1-\frac{1}{c})^{\theta-1}}{1-c^{1-\theta}}
\end{aligned}$$

Summarizing, we have obtained a lower bound to $L_d(s,c)$

$$L_d(s,c) \ \geq \ 1 + \frac{1}{\zeta_C(\theta)\ C(\theta-1)\ (1+C+Cs)^{\theta-1}} \frac{(1-\frac{1}{c})^{\theta-1}}{1-c^{1-\theta}} \tag{7.16}$$

Since Equation 7.1, it is known that $L_d(D,D)$ is a lower bound of $L_h(D)$, thus the following Equation gives a lower bound for a D-ary Huffman code.

$$L_{d(D,D)} \ \geq \ 1 + \frac{(1-\frac{1}{D})^{\theta-1}}{\zeta_C(\theta)\ C(\theta-1)\ (1+C+CD)^{\theta-1}\ (1-D^{1-\theta})} \tag{7.17}$$

7.6 Applying bounds to real text collections

We have obtained the optimal parameters θ and C from Zipf-Mandelbrot's law that approximate the word frequency distribution for all the text collections in our experimental framework. This was made by first choosing suitable values of C (0.75, 1.00, and 1.25), and then applying regression to obtain the best value of θ. Then, assuming Zipf-Mandelbrot's distributions with those parameters, we ran Huffman's algorithm to obtain the average codeword length for a D-ary Huffman code ($L_h(D)$). Finally we also computed the empirical and analytical bounds shown through this chapter.

Table 7.1 compares the redundancy (see Section 2.2) obtained by our bounds and the redundancy of the D-ary Huffman code when they are applied to the largest corpora in our experimental framework. Columns two and three in that table show, respectively, the parameters C and θ used to characterize the word frequencies in each corpus. The lower bounds are shown in the fourth and fifth columns: the fourth one gives the redundancy obtained by the (D, D)-Dense Code ($L_d(D, D) - H_0^D$), and the fifth column shows the redundancy obtained by the analytical lower bound ($[Eq.\ 7.17] - H_0^D$). The sixth column from the table shows the redundancy of the D-ary Huffman code ($L_h(D) - H_0^D$). The last two columns show, respectively, the redundancy obtained by the empirical upper bound ($L_d(s, c) - H_0^D$) and the analytical upper bound ($[Eq.\ 7.11] - H_0^D$).

It can be seen that very tight upper and lower bounds are obtained by running (s, c)-Dense Code and (D, D)-Dense Code respectively. Moreover, it is shown that the analytical bounds work fine, obtaining values very close to the empirical bounds.

7.7 Applying bounds to theoretical text collections

In this section we use theoretical Zipf-Mandelbrot's word frequency distributions and show the behavior of the bounds presented. In Figure 7.2, assuming Zipf-Mandelbrot's law and having $n = 10,000,000$ and $C = 1.0$ we show the bounds obtained for different values of the parameter θ. Both the empirical upper bound (obtained by running (s, c)-Dense Code) and the analytical upper bound are very tight. In fact, the empirical upper bound is so tight that it becomes almost indistinguishable with respect to the D-ary Huffman. In the case of the lower bounds, they are not so tight as the upper bounds are, but they permit predicting

CORPUS	Z-M param.		Redundancy				
	C	θ	An. Low	(D, D)-DC	D-ary Huff	(s, c)-DC	An. Up
ZIFF	0.75	1.462	0.270	0.293	0.313	0.313	0.327
FT93	0.75	1.451	0.262	0.286	0.307	0.308	0.323
FT94	0.75	1.453	0.264	0.287	0.308	0.309	0.323
AP	0.75	1.458	0.267	0.290	0.311	0.311	0.325
FT_ALL	0.75	1.472	0.272	0.297	0.317	0.317	0.326
ALL	0.75	1.465	0.266	0.293	0.314	0.314	0.322
ZIFF	1.00	1.425	0.274	0.295	0.317	0.318	0.337
FT93	1.00	1.416	0.267	0.289	0.312	0.313	0.333
FT94	1.00	1.417	0.267	0.289	0.312	0.313	0.333
AP	1.00	1.422	0.271	0.292	0.315	0.316	0.335
FT_ALL	1.00	1.438	0.277	0.301	0.323	0.323	0.336
ALL	1.00	1.432	0.271	0.297	0.320	0.320	0.332
ZIFF	1.25	1.398	0.275	0.294	0.318	0.319	0.344
FT93	1.25	1.389	0.268	0.287	0.313	0.314	0.340
FT94	1.25	1.391	0.269	0.288	0.314	0.315	0.341
AP	1.25	1.395	0.272	0.291	0.316	0.317	0.342
FT_ALL	1.25	1.412	0.277	0.301	0.325	0.325	0.341
ALL	1.25	1.408	0.272	0.297	0.322	0.323	0.338

Table 7.1: Redundancy in real corpora

the average codeword length of a D-ary Huffman code with little error.

It can be seen that, when low values of θ are used, the redundancy obtained is low. However, differences between the lower and upper bounds and the redundancy of Huffman are maximal. As θ increases, more biased Zipf-Mandelbrot's distributions are generated, and the bounds obtained are tighter than those obtained with lower values of θ.

7.8 Summary

In this chapter we presented how the compression ratio that is obtained by a D-ary Huffman code can be lower and upper bounded by both End-Tagged Dense Code and (s, c)-Dense Code. We first showed that these new techniques are so simple and fast that they can be run to approximate the average codeword length of a D-ary Huffman code, avoiding running the slower Huffman algorithm.

Then, we presented analytical bounds, assuming that Zipf-Mandelbrot's law predicts the word frequency distribution in natural language texts. First, we showed bounds that are based on the *zero*-order entropy. Finally, we provided analytical upper and lower bounds based on (s, c)-Dense Code.

These analytical results are important for compressed Text Databases, because they permit to predict their compressed size from an analytical model of a word

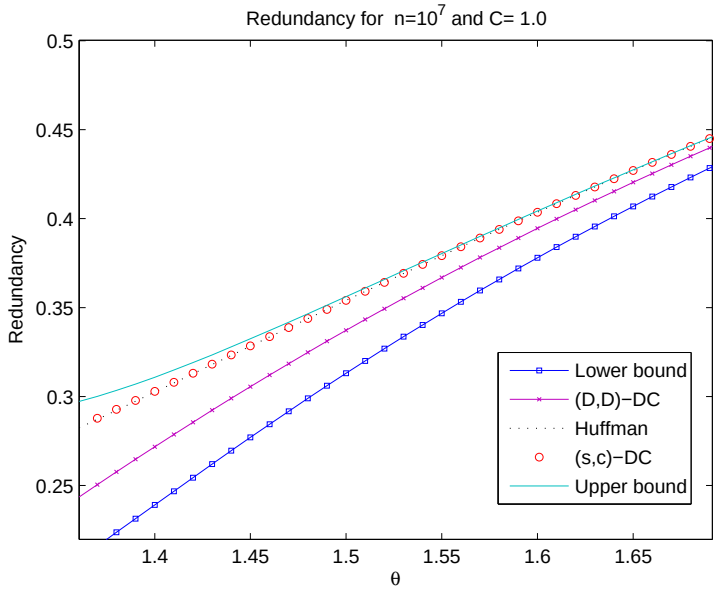


Figure 7.2: Bounds using Zipf-Mandelbrot's law.

distribution in some language.

Part II

Adaptive compression

8

Dynamic text compression techniques

In this chapter, a brief description of the state of the art in *dynamic* text compression techniques is presented. First, the motivation of adaptive versus semi-static techniques is discussed. Then, the two most popular and well-known families of dynamic techniques are described: statistical and dictionary-based compressors.

The first family of dynamic compressors is composed of statistical techniques. The general way of working of dynamic statistical codes is explained, and the two main representatives of this kind of dynamic methods are described in Section 8.2. The first technique is called dynamic character oriented Huffman, and it is described in Section 8.2.1. Dynamic character oriented Huffman is the basis of the dynamic word-based Huffman technique presented in Chapter 9. The other statistical technique, known as dynamic arithmetic compression, is introduced in Section 8.2.2.

In Section 8.3, a predictive technique called Prediction by Partial Matching (PPM) is presented. Finally, Section 8.4 starts with a brief description of *dictionary techniques* and later presents the variants of the Ziv-Lempel family, one of the most widespread compression families, and the basis of common programs such as *gzip*, *compress*, and *arj*.

8.1 Introduction

Transmission of compressed data is usually composed of four processes: *compression*, *transmission*, *reception*, and *decompression*. The first two are carried out by a *sender* process and the last two by a *receiver*. This structure can be seen for example, when a user downloads a zip file from a website. The file, which was previously compressed in the server, is transmitted in compressed form when the user downloads it. Finally, once the transmission has finished, it can be decompressed. This transmission scheme can also be extrapolated to the scenario of a file being compressed or decompressed from disk. In this case, the compressor acts as a sender and the decompressor acts as the receiver.

There are several interesting *real-time* transmission scenarios where the *compression*, *transmission*, *reception*, and *decompression* processes should take place concurrently. That is, the sender should be able to start the transmission of compressed data without preprocessing the whole text, and simultaneously the receiver should start the reception and decompression of the text as it arrives.

Real-time transmission is usually of interest when communicating over a network. In the case of natural language text, this kind of compression can be applied, for example, in the following scenarios:

- Interactive services such as remote talk/chat protocols, where messages are exchanged during the whole communication process. Instantaneous transmission and reception are needed, and they take place in an interactive way. Therefore, it is not feasible to delay the beginning of the transmission of data until the whole text is known.
- Transmission of Web pages. Installing a browser plug-in to handle decompression enables the exchange of compressed pages between a server and a client along time in a more efficient way.
- Wireless communication with hand-held devices with little bandwidth.

Real-time transmission is handled by *dynamic* or *adaptive* compression techniques. As in the case of semi-static methods, these techniques pursue several goals such as good compression ratio, and fast compression and decompression. However, whereas in semi-static techniques the features of direct access and direct search were interesting, in dynamic techniques it is crucial to be able to manage streams of text.

Currently, the most widely used adaptive compression techniques belong to the

Ziv-Lempel family [ZL77, ZL78, Wel84]. When applied to natural language text, however, the compression ratios achieved by Ziv-Lempel are not that good (around 40%). Their advantages are compression speed and, mainly, decompression speed.

Other adaptive techniques like Arithmetic Encoding [Abr63, WNC87, MNW98] or Prediction by Partial Matching (PPM) [BCW84] have proven to be competitive regarding compression ratio. However, they are not time-efficient.

Classic Huffman code [Huf52] is a well-known two-pass method. Making it dynamic was first proposed in [Fal73, Gal78]. This method was later improved in [Knu85, Vit87]. However, being character-based, the compression ratios achieved were not good. It is interesting to point out that the adaptive Huffman-based techniques can be extrapolated to a word-based approach. Such a technique is presented in Chapter 9.

This chapter shows the most common adaptive compression techniques. These techniques will be used in Chapter 11 to experimentally test the dynamic compression techniques developed in this thesis.

8.2 Statistical dynamic codes

Statistical dynamic compression techniques are also called *one-pass*. Symbol frequencies are collected as the text is read, and consequently, the mapping between symbols and codewords is updated as compression progresses. The receiver acts in the same way as the sender. It computes symbol frequencies and updates the correspondence between codewords and symbols each time a codeword is received.

In particular, dynamic statistical compressors model the text using the information about source symbol frequencies, that is, $f(s_i)$ is the number of times that the source symbol s_i appears in the text (read up to now).

In order to maintain the vocabulary up-to-date, dynamic techniques need a data structure to keep all symbols s_i and their frequencies $f(s_i)$ up to now. Such data structure is used by the encoding/decoding scheme, and it is continuously updated during compression/decompression. For each new source symbol, if it is already in the vocabulary, its frequency is increased by 1. If it is not, it is inserted in the vocabulary and its frequency is set to 1. After reading each symbol, the vocabulary is updated and rearranged if necessary, so that the codeword assigned to any source symbol may change.

To let the sender inform the receiver about new source symbols that appear in

```
Sender ( )
(1)  Vocabulary ← {C_new-Symbol};
(2)  Initialize CodeBook;
(3)  while (true)
(4)       read s from the text;
(5)       if s ∉ Vocabulary then
(6)            send C_new-Symbol;
(7)            send s in plain form;
(8)            Vocabulary ← Vocabulary ∪ {s};
(9)            f(s) ← 1;
(10)      else
(11)           send CodeBook(s);
(12)           f(s) ← f(s) + 1;
(13)      Update CodeBook;
```

```
Receiver ( )
(1)  Vocabulary ← {C_new-Symbol};
(2)  Initialize CodeBook;
(3)  while (true)
(4)       receive C;
(5)       if C = C_new-Symbol then
(6)            receive s in plain form;
(7)            Vocabulary ← Vocabulary ∪ {s};
(8)            f(s) ← 1;
(9)       else
(10)           s ← CodeBook^-1(C);
(11)           f(s) ← f(s) + 1;
(12)      output s;
(13)      Update CodeBook;
```

Figure 8.1: Sender and receiver processes in statistical dynamic text compression.

the text, a special source symbol *new-Symbol* (whose frequency is zero by definition) is always held in the vocabulary. The sender transmits the *new-Symbol* codeword each time a new symbol arises in the source text. Then, the sender encodes the source symbol in plain form (e.g., using ASCII code for words) so that the receiver can insert it in its vocabulary.

Figure 8.1 depicts the sender and receiver processes, highlighting the symmetry of the scheme. *CodeBook* stands for the mapping between symbols and codewords, and permits to assign codewords to source symbols or vice versa. Note that *new-Symbol* is always the least frequent symbol of the *CodeBook*.

8.2.1 Dynamic Huffman codes

In [Fal73, Gal78], an adaptive character-oriented Huffman code algorithm was presented. It was later improved in [Knu85], being named *FGK* algorithm. *FGK* is the basis of the UNIX *compact* command.

FGK maintains a Huffman tree for the source text already read. The tree is adapted each time a symbol is read to keep it optimal. It is maintained both by the sender, to determine the code corresponding to a given source symbol, and by the receiver, to do the opposite.

Thus, the Huffman tree acts as the *CodeBook* of Figure 8.1. Consequently, it is initialized with a unique special node called *zeroNode* (corresponding to *new-Symbol*), and it is updated every time a new source symbol is inserted in the vocabulary or when a frequency is increased. The codeword for a source symbol corresponds to the path from the tree root to the leaf corresponding to that symbol. Any leaf insertion or frequency change may require reorganizing the tree to restore its optimality.

The main challenge of Dynamic Huffman is how to reorganize the Huffman tree efficiently upon leaf insertions and frequency increments. This is a complex and potentially time-consuming process that must be carried out both by the sender and the receiver.

The basis of the *FGK* algorithm is the *sibling property* defined by Gallager in [Gal78].

Definition 8.1 *A binary code tree has the sibling property if each node (except the root) has a sibling and if all nodes can be listed in decreasing weight order, with each node adjacent to its sibling.*

Gallager also proved that a binary prefix code is a Huffman code iff the code tree has the sibling property.

Using the *sibling property*, the main achievement of *FGK* is to ensure that the tree can be updated by doing only a constant amount of work per node in the path from the affected leaf to the tree root. Calling $l(s_i)$ the path length from the leaf of source symbol s_i to the root, and $f(s_i)$ its frequency, the overall cost of the algorithm *FGK* is $\sum f(s_i)l(s_i)$, which is exactly the length of the compressed text measured in number of target symbols.

In [Vit87] an improvement upon *FGK* denominated *algorithm* Λ was presented.

An implementation of Λ can be found in [Vit89]. The main difference with respect to *FGK* is that algorithm Λ uses a different method to update the tree, which not only minimizes $\sum f(s_i)l(s_i)$ (compressed text length) but also the external path length ($\sum l(s_i)$), and the height of the tree ($\max l(s_i)$). Moreover, algorithm Λ reduces to 1 the number of exchanges in which a node is moved upwards in the tree during an update of the tree. Although these improvements do not modify the complexity of the whole algorithm, they give algorithm Λ advantage in compression ratio and speed over *FGK*, and even over static Huffman for small messages. Results presented in [Vit87] show that adaptive Huffman methods are directly comparable to classic Huffman in compression ratio.

8.2.2 Arithmetic codes

Arithmetic coding was first presented in the sixties in [Abr63]. Being statistical, it uses the probabilities of the source symbols in order to obtain compression.

Distinct models can be used to calculate, for a given context, the probability of the next source symbol. Therefore static, semi-static, and adaptive arithmetic codes are available. The key idea of this technique is to represent a sequence of source symbols using a **unique** real number in the range [0,1). As the message to be encoded becomes larger, the interval needed to represent it becomes narrower, and therefore, the number of bits needed to represent it grows.

Basically an arithmetic encoder works with a list of the n symbols of the vocabulary and their probabilities. The initial interval is [0,1). When a new source symbol is processed, the interval is reduced in accordance with the current symbol probability and the interval becomes a narrower range that represents the input sequence of symbols already processed.

We present in Example 8.1 a semi-static arithmetic compressor to explain how arithmetic compression works. Note that making it dynamic consists only of adapting the frequency of the source symbols each time one of them is processed.

Example 8.1 Let us compress the message "AABC! " using a semi-static model. In the first phase, the compressor creates the vocabulary, which is composed of four symbols: 'A', 'B', 'C' and '!'. Their frequencies are: 0.4, 0.2, 0.2 and 0.2 respectively. Therefore, in the initial state, any number in the interval [0, 0.4) represents symbol 'A', and intervals [0.4, 0.6), [0.6, 0.8) and [0.8, 1) represent symbols 'B', 'C', and '!' respectively.
Since the first symbol to encode is 'A', the interval [0,1) is reduced to [0, 0.4).

Next possible sub-intervals are [0, 0.16), [0.16, 0.24), [0.24, 0.32), and [0.32, 0.4). They would represent the sequences 'AA', 'AB', 'AC' or 'A!'. Figure 8.2 represents graphically the intervals of the whole process. Since the next symbol is again 'A', the current working-interval is reduced to [0, 0.16). Note that the size of this interval depends on the probability of the sequence encoded; that is: $0.4 \times 0.4 = 0.16$.
To encode the next source symbol 'B', the new interval is reduced to [0.064, 0.096), because the sequence 'AAB' has probability of $0.032 = 0.096 - 0.064$.
After processing 'C', the range becomes [0.0832, 0.0896), and the probability associated to 'AABC' is 0.0064.
Finally, the possible sub-intervals are [0.0832, 0.08576), [0.08576, 0.08704), [0.08704, 0.08832), and [0.08832, 0.0896). Since '!' was the last symbol of the vocabulary, any number in the interval [0.08832, 0.0896) represents the message 'AABC!'. Therefore, the encoder generates the number that can be encoded with less bits inside that interval. □

The decompressor only has to know the vocabulary used, the probabilities of the source symbols and the number of symbols transmitted. From the compressed data, it can detect the intervals used in the encoding phase and from these intervals, it recovers the source symbols.

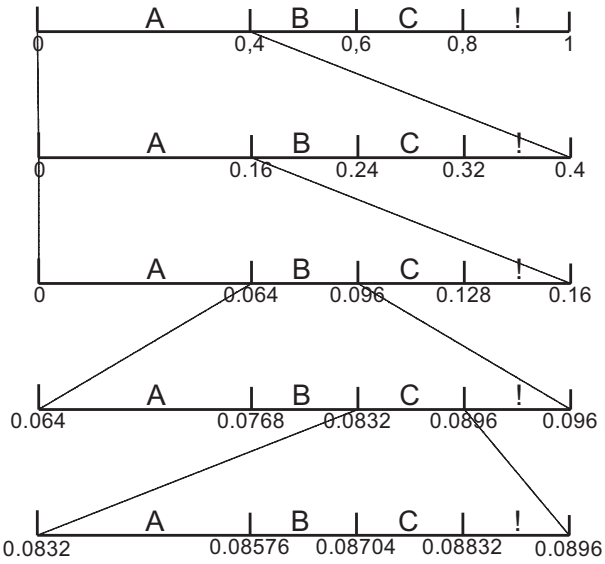


Figure 8.2: Arithmetic compression for the text AABC!.

In general, arithmetic compression improves Huffman compression ratios. When

static or semi-static models are used, compression and decompression speed are not competitive with respect to Huffman-based techniques. Moreover, they have the disadvantage that decompression and searches cannot be performed directly in the compressed text because it is represented by a single number, so decompression is mandatory before a search can be performed. As a result, arithmetic coding is not useful in environments where text retrieval capabilities are needed. However, it becomes a good alternative to adaptive Huffman codes. In [WNC87], an adaptive character based arithmetic technique is compared with the *UNIX compact* algorithm (which is based on FGK dynamic Huffman). In this case arithmetic encoding is faster in both compression and decompression processes, and it also achieves better compression ratio.

Several modifications of the basic arithmetic algorithm improving its performance and/or using distinct models have been made. In [WNC87], Witten, Neal, and Cleary presented an arithmetic encoder based on the use of integer arithmetic. In [MNW98], Moffat et al. made improvements focused on avoiding using multiplications and divisions that could be replaced with faster shift/add operations. Moreover, the code from [CMN+99] (based on [MNW98]) is of public domain and it has been used in our tests.

8.3 Prediction by Partial Matching

Prediction by Partial Matching technique (PPM) was first presented in 1984 by Cleary and Witten [CW84].

PPM is an adaptive statistical data compression technique based on context modelling and prediction. Basically, PPM uses sequences of previous symbols in the source symbol stream to predict the frequency of the next symbol, and then it applies an arithmetic technique [WNC87, MNW95] to encode that symbol using the predicted frequency.

PPM is based on using the last m characters from the input stream to predict the probability of the next one. This is the reason why it is called a *finite-context model of order m*. That is, a finite-context model of order 2 will use only the two previous symbols to predict the frequency of the next symbol.

PPM combines several finite-context models of order m, in such a way that m takes the values from 0 to M (M is the maximum context length). For each finite-context model, PPM takes account of all m-length sequences S_i that have previously appeared. Moreover, for each of those sequences S_i, all characters that

have followed S_i, as well as the number of times they have appeared, are kept. The number of times a character followed an m-length sequence is used to predict the probability of the incoming character in the model of order m. Therefore, for each of the finite-context models used, a separate predicted probability distribution is obtained.

The maximum context length (that is, the length of a sequence of symbols that are considered in the highest-order model) is usually 5. It was shown [CW84, Mof90] that increasing the context length beyond $5-6$ symbols does not usually improve compression ratio.

The $M+1$ probability distributions are blended into a single one, and arithmetic coding is used to encode the current character using that distribution. In general, the probability predicted by the highest-order model (the M-order model) is used. However, if a novel character is found in this context (no M-length sequence precedes the new character), then it is not possible to encode the new character using the given M-order model, and it is necessary to try the $(M-1)$-order model. In this case, an *escape symbol* is transmitted to warn the decoder that a change from an M to an $(M-1)$-order model occurs. The process continues until it reaches a model where the incoming symbol is not novel (and then that symbol can be encoded with the frequency predicted by the model). To ensure that the process always finishes, a (-1)-order model is assumed to exist. This bottom-level model predicts all characters s_i from the source alphabet (Σ) with the same probability, that is $p(s_i) = \frac{1}{|\Sigma|}$.

Notice that each time a model shift (from m to $m-1$) occurs due to a novel symbol in the context model of order m, the probability given to the *escape symbol* in the m-order model needs to be combined with the probability that the $(m-1)$-order model assigns to the symbol being encoded.

We call ω the symbol whose probability is being predicted, $p_m(\omega)$ the probability that the m-order model assigns to ω and e_m the probability assigned to the *escape symbol* by the m-order model. Two situations can arise:

1. ω can be predicted by the M-order model. In this case ω is encoded using the probability $p(\omega) = p_M(\omega)$.
2. ω can be predicted by an m-order model ($m < M$). Then the probability used to encode ω is $p(\omega) = p_m(\omega) \times \prod_{l=m+1}^{M} e_l$.

Distinct methods can be used to assign probabilities both to the *escape symbol* and to a novel source symbol. For example, PPMA [CW84] uses a method called A,

which makes $p_m(\omega) = \frac{c_m(\omega)}{1+c_m}$ and $e_m = \frac{1}{1+c_m}$, where $c_m(\omega)$ is the number of times that the character ω appeared in the m-order model, and c_m is the total count of characters that were first predicted by an m-order model. PPMC [Mof90] is the first popular PPM variant because PPM algorithms require a significant amount of memory and previous computers were not powerful enough. PPMC uses *method C* to assign probabilities. It sets $e_m = \frac{d_m}{c_m}$, and $p_m(\omega) = \frac{(c_m - d_m) \times c_m(\omega)}{c_m}$, where d_m is the number of distinct characters that were first predicted by the m-order model.

Choosing a method to assign probabilities to the *escape symbol* constitutes an interesting problem called *the zero-order frequency problem*. Distinct methods have been proposed: *Methods A and B* [CW84], *method C* [Mof90], *method D* [How93] and *method X* [WB91]. A description and a comparison of the those available methods can be found in [MT02].

Recent PPM implementations obtain good compression results. As shown in [CTW95], PPMC uses 2.48 bits per character (bpc) to encode the Calgary corpus and another variant PPM* (or PPMX)[CTW95] needs only 2.34 bpc (less than 30% in compression ratio). However, compression and decompression speed are not so good (5 times slower than *gzip* in compression).

Nowadays, the main competitor of PPM in compression ratio is *bzip2*. In [WMB99], it is shown that PPM and *bzip2* compression ratio and compression speed are quite similar (with a small advantage to PPM). However, *bzip2* is 3 times faster than PPM in decompression. These reasons (similar compression, but worse decompression) and the fact that *bzip2* has become a well-known and widespread compression technique, led us to use *bzip2* instead of PPM in our tests.

8.4 Dictionary techniques

Dictionary techniques do not take into account the statistics of the number of occurrences of symbols in a text. They are based on building a dictionary where substrings are stored, in such a way that each substring is assigned a codeword (usually the codeword is given by its position in the dictionary). Each time a substring is read from the source stream, it is searched in the dictionary, and the substring is substituted by its codeword. Compression is achieved because the substituted substrings are usually larger than their codewords.

Dictionary techniques are the most commonly used compression techniques. In fact, the Ziv-Lempel family [ZL77] is the most representative Dictionary technique, and its two main variants *LZ77* [ZL77] and *LZ78* [ZL78] are the base of several

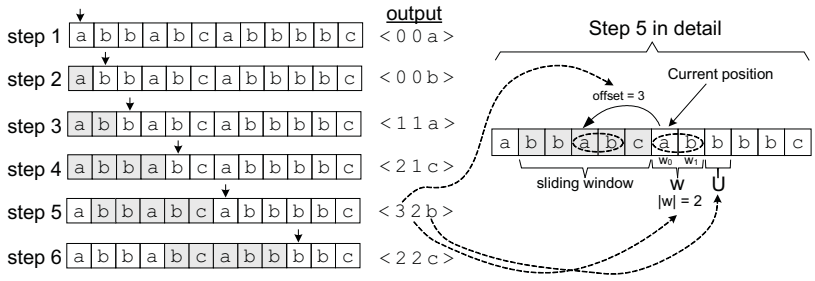


Figure 8.3: Compression using LZ77.

common compression programs. For example, programs such as *gzip*, *pkzip* and *arj* are based on *LZ77*, and *LZ78* is the base of the *UNIX* program *compress*.

8.4.1 LZ77

This is the first technique that Abraham Lempel and Jacob Ziv presented in 1977 [ZL77]. LZ77 uses the dictionary strategy. It has a dictionary that is composed of the n last characters already processed (n is a fixed value) and is commonly called the *sliding window*.

Compression starts with an empty *sliding window*. In each step of the compression process, the largest substring w that already appeared in the *window* is read. That is, characters $w_0, w_1, ..., w_k$ after the *window* are read as long as the substring $w = w_0, w_1, ..., w_k$ can be found in the *window*. Assume that U is the next character after w. Then the compressor outputs the triplet $<$ *position*, *length*, $U >$, and the *window* is slid by $k + 1$ positions. If $w = \varepsilon$ (an empty string) then the triplet $< 0,\ 0,\ U >$ is output. The *position* element of the triplet represents the backwards offset with respect to the **end** of the *window* where w appears, and the *length* element corresponds to the size of the w substring (that is, k).

Example 8.2 Let us compress the text "`abbabcabbbbc`" using *LZ77* technique assuming a *sliding window* of only 5 bytes. The process, summarized in Figure 8.3, consists of the following six steps:

1. The *window* starts with no characters, so `'a'` is not in the dictionary and $< 0, 0, a >$ is output. Then, the window is slid by 1 character to the right, so the *window* will contain ⟨-,-,-,-,'a'⟩.

2. Next `'b'` is read, it is not in the dictionary and $< 0, 0, b >$ is output. Then the window is slid again by 1 position. Hence the new *window* contains ⟨-,-,-,'a','b'⟩.

3. A new `'b'` is read. It was already in the last position of the *window*, but the string 'ba' was not in the vocabulary yet. Therefore $w =$ `'b'`, $U =$ `'a'` and $< 1, 1, a >$ is output. The window is slid by $|w| + 1 = 2$ positions, so it becomes ⟨-,'a','b','b','a'⟩.

4. The next substring is `'b'`, but `'bc'` does not appear in the *sliding window*, so $w =$ `'b'`, $U =$ `'c'` and $< 2, 1, c >$ is output. After sliding the *window* by $|w| + 1 = 2$ positions, it contains ⟨'b','b','a','b','c'⟩.

5. The process continues reading `'abb'`, such as `'ab'` is a prefix in the *window*, and $< 3, 2, b >$ is output. The new *window* contains ⟨'b','c','a','b','b'⟩.

6. Finally, since `'bb'` is a new recognized substring but `'bbc'` does not appear in the *sliding window*, $< 2, 2, c >$ is output. □

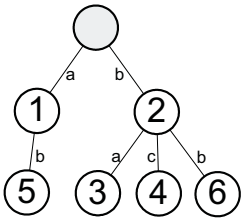
During decompression the *window* holds the last decoded elements. Given a triplet $< position, length, nextChar >$, the decompressor outputs *length* characters starting *position* elements before the end of the *window* and finally it also outputs *nextchar*. Notice that the vocabulary is quite small (it can be kept in cache in a current processor), and the decoding process implies only one array lookup. Therefore, decompression is very fast.

In general, a minimum substring size (usually 3 characters) is used to avoid the substitution of small prefixes. Moreover, the length of the *sliding window* has a fixed value. A higher value permits finding larger substrings in the *sliding window*. However, that implies that a larger pointer will also be needed to represent the *position* element of the triplet. In general, the *position* element is represented by 12 bits (hence the *sliding window* has 4096 bytes at most), and 4 bits are used for the *length* element. That is, 2 bytes are needed to represent both *position* and *length*.

8.4.2 LZ78

Instead of a window that contains the last processed text, *LZ78* technique [ZL78] builds a dictionary that holds all phrases[1] recognized in the text. This dictionary is efficiently searched via a *trie* data structure in such a way that a node l_i in the *trie*

[1] A phrase is just a substring of the source text.



Trie structure

step	input	output	Dictionary
1	a	(0,a)	entry_1 = "a"
2	b	(0,b)	entry_2 = "b"
3	ba	(2,a)	entry_3 = "ba"
4	bc	(2,c)	entry_4 = "bc"
5	ab	(1,b)	entry_5 = "ab"
6	bb	(2,b)	entry_6 = "bb"
7	bc	(4,ε)	–

Figure 8.4: Compression of the text "abbabcabbbbc" using LZ78.

stores a pointer i to a dictionary entry (entry_i), and the path from the root of the *trie* to node l_i spells out the letters of entry_i. That is, the labels in the branches of the trie correspond to the letters of $entry_i$.

LZ78 has simple encoding and decoding procedures. Basically encoding consists of:

i) From the current position in the text, the longest entry in the vocabulary (entry_i) that matches with the following characters in the text is found.

ii) The pair (i, U) is output (where U is the character that follows entry_i in the text).

iii) The new phrase (entry_i+U) is appended to the dictionary.

Figure 8.4 explains how to compress the text "`abbabcabbbbc`" using *LZ78* technique.

Decoding is done in a symmetric form to encoding, this time the trie has to be traversed upwards, and a vector representation is preferable.

Compression in *LZ78* is faster than in *LZ77*, but decompression is slower because the decompressor needs to store the phrases. However, this technique results interesting and a variant of *LZ78* called *LZW* is widely used.

8.4.3 LZW

LZW is a variant of *LZ78* proposed by Welch in 1984 [Wel84]. *LZW* is widely used: the Unix *compress* program and the *GIF* image format use this technique to achieve compression. The main difference of *LZW* with respect to *LZ78* is that

input	next character	output	Dictionary
			entry_0 = "a"
			entry_1 = "b"
			entry_2 = "c"
a	b	0	entry_3 = "ab"
b	b	1	entry_4 = "bb"
b	a	1	entry_5 = "ba"
ab	c	3	entry_6 = "abc"
c	a	2	entry_7 = "ca"
ab	b	3	entry_8 = "abb"
bb	b	4	entry_9 = "bbb"
b	c	1	entry_{10} = "bc"
c	ε	2	–

Table 8.1: Compression of "abbabcabbbbc", Σ={a, b, c}, using LZW.

LZW only outputs a list of pointers, while *LZ78* outputs also characters explicitly. *LZW* avoids outputting those characters explicitly by initializing the dictionary with phrases that include all the characters from the source alphabet (for example, the 128 ASCII values) and taking the last character as the first of the next phrase.

The encoding process starts with the initial dictionary. From the input, the longest phrase $w_0 w_1 \dots w_k$ that exists in the dictionary is searched (note that, since all characters belong to the dictionary, phrases are always found in the dictionary). Suppose that the entry in the dictionary which corresponds to that phrase is i (that is, $phrase_i = w_0 w_1 \dots w_k$ and the phrase $w_0 w_1 \dots w_k w_{k+1}$ did not appear in the dictionary previously). Then i is output and a new phrase, formed as the concatenation of $phrase_i$ and the next character in the input w_{k+1}, is added to the dictionary (that is, the phrase $w_0 w_1 \dots w_k w_{k+1}$). Then the process iterates reading the next characters $w_{k+2} w_{k+3} \dots w_k w_{k+m}$ from the input and searching again for the longest phrase in the dictionary which contains $w_{k+1} w_{k+2} w_{k+3} \dots w_{k+m}$. An example describing the whole compression process is shown in Table 8.1.

Since a maximum dictionary size has to be defined, some decision has to be taken when all the available entries in the dictionary are already used:

- Do not permit more entries and continue compression with the current dictionary.
- Use a *least-recently-used* policy to discard entries when a new entry has to be added.

- Delete the whole dictionary and start again with an empty one (which has only the initial entries, that is,the symbols of the alphabet).
- Continue monitoring the compression ratio, and drop the dictionary when compression decreases.

8.4.4 Comparing dictionary techniques

Two Ziv-Lempel based techniques are widely used. The most used is *gzip* (based on LZ77) and the second is *compress* (based on LZW). As it is shown in [WMB99], *gzip* achieves better compression ratio (around 35-40%) and decompression speed than *compress* (compression ratio around 40%). However, *compress* is a bit faster in compression. *Gzip* was used in our tests as a representative of dictionary-based techniques. We decided to use *gzip* instead of *compress* because it has better compression ratio in natural language texts.

8.5 Summary

In this chapter, the state of the art in dynamic text compression techniques has been reviewed. Special attention was paid to statistical methods such as Huffman-based ones. Arithmetic compression, as well as a predictive model such as Prediction by Partial Matching technique, were shown. Finally, a brief review of Ziv-Lempel based compressors, the most commonly used compression family, was also presented.

9

Dynamic byte-oriented word-based Huffman code

A dynamic byte-oriented word-based Huffman code is presented in this chapter. This is a practical contribution of this thesis. Even though we developed and implemented this technique in order to compare it against the Dynamic End-Tagged Dense Code and Dynamic (s, c)-Dense Code, presented in Chapters 10 and 11 respectively, the results achieved in compression ratio, as well as in compression/decompression speed, make this a competitive adaptive technique, and hence a valuable contribution by itself.

We start with the motivation of developing a word-based byte-oriented Huffman code. Next, some properties and definitions on the code are shown. The technique is described in Section 9.3. Later, the data structures used and the update algorithm that enables maintaining a well-formed Huffman tree dynamically are shown. In Section 9.6, some empirical results compare our Dynamic word-based Huffman Code against the well-known character-based approach. Finally, the new dynamic code is also compared against its semi-static counterpart, the Plain Huffman Code.

9.1 Motivation

We have already presented in Section 8.2.1 some variants of dynamic Huffman-based techniques. However, those methods are character-oriented, and thus their

compression ratios on natural language is poor (around 60%).

It was also shown in Section 4.2 that new semi-static word-based Huffman compression techniques for natural language appeared in the last years. The first ones [Mof89] are bit-oriented techniques and achieve compression ratios around 25%. Tagged Huffman and Plain Huffman codes [MNZBY00] use a byte- rather than bit-oriented coding alphabet, what speeds up decompression and search at the expense of a small loss in compression ratio (under 5 percentage points).

Two-pass codes, unfortunately, are not suitable for real-time transmission. Hence, developing a fast adaptive compression technique with good compression ratio for natural language texts is a relevant problem.

In this chapter, a dynamic word-based byte-oriented Huffman method is presented. It retains the advantages of the semi-static word-based techniques (compression ratio around 30%-35% in large texts) and the real-time facilities of the dynamic character-based bit-oriented Huffman techniques. Moreover, being byte- rather than bit-oriented, compression and decompression speed are competitive.

9.2 Word-based dynamic Huffman codes

We developed a word-based byte-oriented version of the algorithm *FGK* which satisfies two necessary conditions: *i)* only one pass over the text is performed, *and ii)* it is needed to maintain a well-formed 2^b-ary Huffman tree during the compression process.

As the number of different text words is much larger than the number of distinct characters, several challenges arise to manage such a large vocabulary. The original *FGK* algorithm pays little attention to these issues because of its underlying assumption that the source alphabet is not very large.

For example, our sender must maintain a hash table that permits fast searching for a word s_i, in order to obtain its corresponding tree leaf and its current frequency. In the character-oriented approach, this can be done by simply using an array indexed by character.

However, the most important difference between our word-based version and the original *FGK* is the fact that our codewords are byte- rather than bit-oriented. Although this necessarily implies some loss in compression ratio, it gives a decisive advantage in efficiency. Recall that the algorithm complexity corresponds to the number of target symbols in the compressed text. A bit-oriented approach requires

time proportional to the number of bits in the compressed text, while ours requires time proportional to the number of bytes. Hence byte-coding could be up to 8 times faster.

Being byte-oriented implies that each internal node can have up to 2^b children in the resulting Huffman tree, instead of 2 as in a binary tree. This requires extending *FGK* algorithm in several aspects. In particular, some parent/child information that is made implicit by an appropriate node numbering, must be made explicit when the tree arity exceeds 2. So each node must store pointers to its first and last child. Also, the process of restructuring the tree each time a source symbol is processed, is more complex in general.

The *sibling property* (Definition 8.1) was redefined for its use in a byte-oriented Huffman code as follows:

Definition 9.1 *A 2^b-ary code tree has the sibling property if each node (except the root and the zeroNode and its siblings), has $2^b - 1$ siblings, and if all nodes can be listed in decreasing frequency order, with each node adjacent to its siblings.*

Recall that the *zeroNode* is a special zero-frequency node that is used in dynamic codes to introduce those symbols that have not yet appeared.

Intuitively, it can be seen that in a 2^b-ary well-formed Huffman tree all internal nodes have 2^b children. However the parent of the *zeroNode* can have less than 2^b children. In fact, if n is the number of symbols (leaves) in a 2^b-ary Huffman tree (including the *zeroNode*), the parent of the *zeroNode* has exactly R child-nodes (see Section 4.2).

Note that the *zeroNode* and its $R - 1$ siblings are the least frequent leaf nodes of the tree.

Property 9.1 *To achieve a dynamic word-based, byte-oriented Huffman code it is only needed to maintain two conditions:*

- *All nodes of the Huffman tree (both internal and leaf nodes) remain sorted by frequency. In this ranking the root is the most frequent node, and the zeroNode is the least frequent node.*
- *All the siblings of a node remain adjacent.*

To maintain the conditions needed by Property 9.1, a node-numbering method was considered. It ranks nodes in the following way:

- Nodes in the top levels of the tree precede nodes in the bottom levels.
- For a given level of the tree, nodes are ranked left-to-right. Therefore the left-most node of a level precedes all nodes in that level.

9.3 Method overview

The compressor/sender and decompressor/receiver algorithms follow the general guidelines shown in Figure 8.1. Considering that general scheme, the main issue that has to be taken into account is how to maintain the *Codebook* up to date, after insertions of new words, or when the frequency of a word is increased.

In this case, the *Codebook* is a 2^b-ary Huffman tree for the words processed up to a given moment of the compression/decompression process.

This Huffman tree is used by both the encoder and the decoder to handle the *encoding* of a word and also the *decoding* of a codeword. In the encoding process, a codeword is given by the path from the root of the tree to the leaf node where that word is placed. The process of decoding a codeword starts in the root of the tree, and each byte value in the codeword specifies the child node that has to be chosen in the next step in a top-down traversal of the tree. A word is recognized each time a leaf node is reached.

Once a encoding/decoding of a word s_i has been carried out, the tree has to be updated. If s_i was already in the tree, the frequency of the leaf node q representing s_i has to be increased, taking into account that the order by frequency of nodes must be maintained. To achieve this, the first node t in the node-numbering such that $frequency_q = frequency_t$ is located. Next step is to exchange nodes q and t. After that, q's frequency can be increased (without altering the order of nodes). Finally the increase of frequency has to be promoted to q's parent. The process continues until the root of the tree is reached.

If s_i was not in the Huffman tree, a new node q representing s_i has to be added (with $frequency = 1$). Two situations can occur: *i)* if $zeroNode$ has less than $2^b - 1$ siblings then q is added as a new sibling of the $zeroNode$. *ii)* If the $zeroNode$ has $2^b - 1$ siblings then a new internal node p is created in the position of $zeroNode$ and both q and $zeroNode$ are set as children of p. In both cases the increase of frequency has to be promoted to the ancestors of q.

Figure 9.1 shows how a 4-ary Huffman tree is maintained by the adaptive process when the sequence "aaaaaaaaabbbbbbbbcddddddccccceeefg" is transmitted.

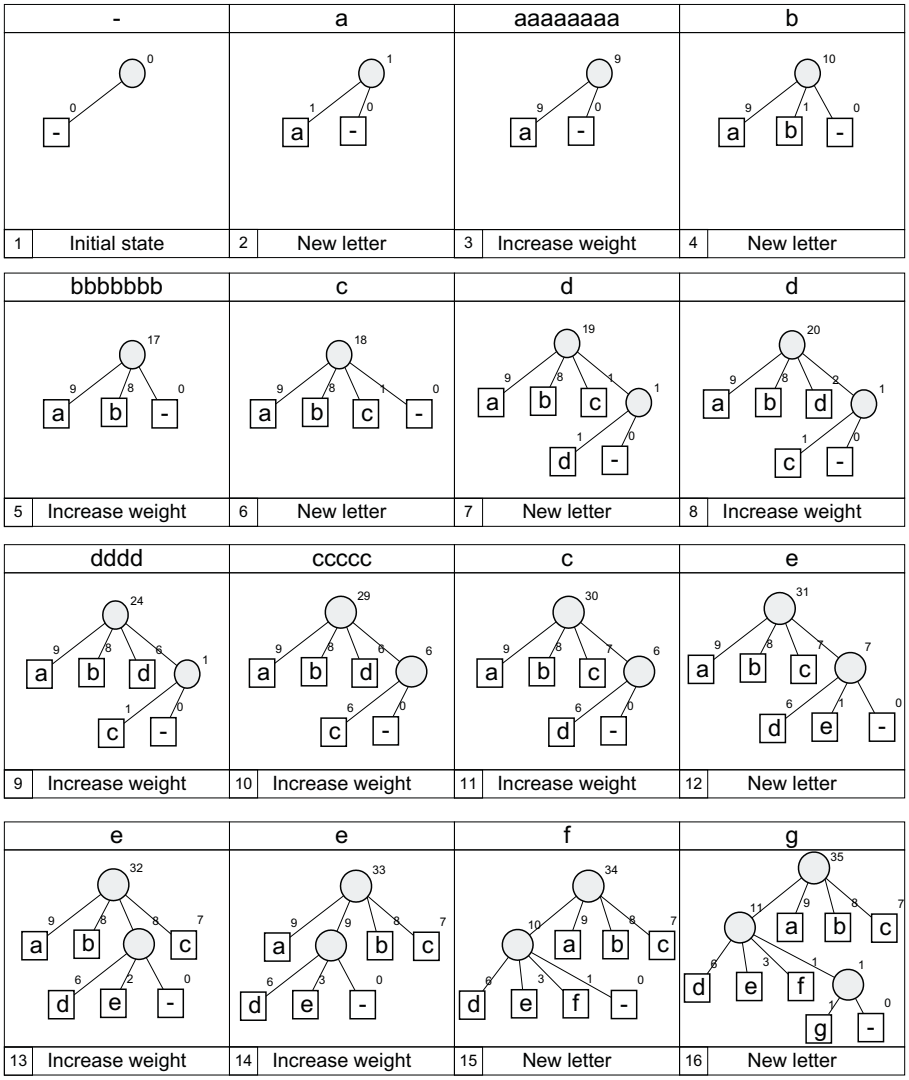


Figure 9.1: Dynamic process to maintain a well-formed 4-ary Huffman tree.

The first box shows the initial state of the Huffman tree. The only nodes in the tree are the *zeroNode* and the *root* node. In step 2, a new node 'a' is inserted into the tree as a sibling of the *zeroNode* (as the parent of the *zeroNode* has enough space to hold 'a', no extra internal nodes have to be added). In step 7, node 'd' is added. As the *zeroNode* has already $2^b - 1 = 3$ siblings, a new internal node is created. In the eighth box, increasing the frequency of node 'd' implies an exchange with node 'c' (needed to maintain the ordered list of nodes). In step 13, the increase of frequency of node 'e' cause an exchange between the parent of 'e' and node 'c'.

9.4 Data structures

In this section, the data structures that support this code are presented. Two main parts arise: *i)* A data structure that supports the Huffman tree, and enables fast lexicographical searches (given a word, find the node representing it in the tree); and *ii)* A *blocks* data structure that, given a frequency value x, enables finding efficiently the first node q whose frequency is x in the ordered list of nodes.

9.4.1 Definition of the tree data structures

Assume that the maximum number of distinct words that will be encoded is n. That is, n is the maximum number of leaf nodes of the Huffman tree and the number of internal nodes is at most $\lceil n/(2^b - 1) \rceil$. Let us call $N = n + \lceil n/(2^b - 1) \rceil$, the maximum number of nodes in the tree.

The Huffman tree is represented by three different data structures: *i)* a *node index* that enables the management of both leaves and internal nodes independently of their type, *ii)* an *internal nodes* data structure, and *iii)* a *hash table* that holds the words of the vocabulary (and hence the leaf nodes of the tree). Two more variables, $zeroNode$ and $maxInternalNode$, handle respectively the first free position in the *node index* and the first free position in the *internal nodes* data structure. Figure 9.2 presents all those data structures and their relationships.

Node Index

As shown, this data structure keeps up to N nodes (both internal and leaf nodes). It consists of four arrays, all of them with N elements:

- nodeType$[q] = 'I'$, $1 \leq q \leq N$ iff the node with the q^{th} highest frequency of the Huffman tree is an internal node. If it is a leaf node then nodeType $[q] = 'L'$.
- freq$[q] = w$, $1 \leq q \leq N$. Depending on the value nodeType$[q]$:
  - If nodeType$[q] = 'L'$ then freq$[q] = w$ indicates that the number of occurrences of the node with the q^{th} highest frequency in the tree is w.
  - If nodeType$[q] = 'I'$ then freq$[q] = w$ represents the summation of the frequencies of all the leaf nodes located in the subtree whose root is the node with the q^{th} highest frequency in the tree.
- parent$[q] = j$, $1 \leq q \leq N$ and $1 \leq j \leq \lceil n/(2^b - 1) \rceil$, iff the internal node with the j^{th} highest frequency is the parent of node q. In the case of the root of the tree, that is, the first node in the node index, $parent[1] = 1$.
- relPos$[q] = j$, $1 \leq q \leq N$. Depending on the value nodeType$[q]$:
  - If nodeType$[q] = 'I'$ then relPos$[q] = j$, $1 \leq j \leq \lceil n/(2^b - 1) \rceil$ means that the node with the q^{th} highest frequency of the tree corresponds to the internal node that is stored in the j^{th} position of the *internal nodes* data structure.
  - If nodeType$[q] = 'L'$ then relPos$[q] = j$, $1 \leq j \leq nextPrime(2n)$ means that the node with the q^{th} highest frequency of the tree corresponds to the leaf node stored in the hash table in the j^{th} position.

Internal nodes data structure

It stores the data that algorithms need for the internal nodes. As shown, the maximum number of internal nodes is $\lceil n/(2^b - 1) \rceil$. Three vectors are used:

- minChild$[i] = q$, $1 \leq i \leq \lceil n/(2^b - 1) \rceil$, $1 \leq q \leq N$, iff the first (most frequent) child of internal node i is held in the q^{th} position of the *node index*.
- maxChild$[i] = q$, $1 \leq i \leq \lceil n/(2^b - 1) \rceil$, $1 \leq q \leq N$, iff the last (least frequent) child of the internal node i is located in the q^{th} position of the *node index*. Note that it always holds that $maxChild[i] - minChild[i] = 2^b - 1$ (except if i is the parent of the *zeroNode*, where the number of children of i can be smaller than 2^b).
- inodePos$[i] = q$, $1 \leq i \leq \lceil n/(2^b - 1) \rceil$, $1 \leq q \leq N$. It means that the i^{th} internal node is the q^{th} node of the *node index*.

Hash table to hold leaf nodes

It is used to store the words of the vocabulary so that they can be located quickly. The hash table has two vectors.

- word$[i] = w$, $1 \leq i \leq nextPrime(2n)$. That is, a word w, is in position i in the hash table. This happens if and only if: *i)* $i = f_{hash}(w)$ and $words[i] = Null$ before adding w into the tree, or *ii)* $i' = f_{hash}(w)$, $words[i'] \neq w$ and $i = solveCollision(i', w)$.

  We are using closed-hashing. The hash function uses a fast bit-wise approach[1]. The function $solveCollision(i', w)$ repeats the instructions

  $$i \leftarrow ((i' + jump) \ mod \ ht\_size); \quad i' \leftarrow i;$$

  until i is a free slot in the hash table, where $nextPrime(2n) = ht\_size$ and $jump$ is a small prime number (101 in this case).

- lnodePos$[i] = q$, $1 \leq i \leq nextPrime(2n)$, $1 \leq q \leq N$. It means that the word in the position i in the hash table corresponds to the q^{th} node of the *node index*.

Figure 9.2 shows how the tree data structures are used assuming a 4-ary tree. In the right part, the tree of step 12 in Figure 9.1 is shown. On the left, those vectors that represent the tree are shown.

Notice that the tree data structures just defined are enough to maintain all nodes in the Huffman tree sorted by frequency. In fact, the first position in the node index stores the root of the tree (the most frequent node), the second position holds the most frequent node among the remainder ones, and so on. However, with only those data structures it is time-expensive to find the *first* node of a given frequency in the *node index*, since either a binary or a sequential search should be made. Remember that, as explained in Section 9.3, finding the *first* node with a given frequency is needed, at least once, when updating the tree. In order to improve the performance of such search process the following *list of blocks* structure is used.

9.4.2 List of blocks

Using the idea pointed in [Knu85, Vit87] nodes with the same frequency are joined into *blocks* and a list of *blocks* is maintained sorted in decreasing order of frequency.

[1] The Justin Zobel's bit-wise hash function used is available at the website http://www.cs.rmit.edu.au/~hugh/zhw-ipl.html.

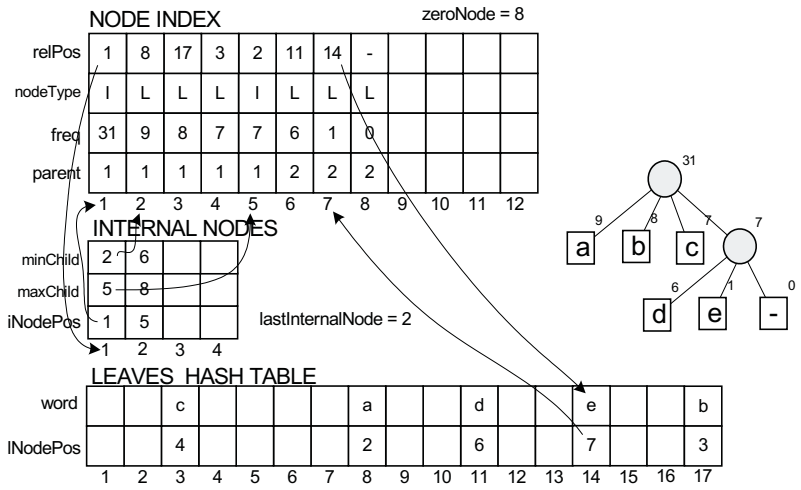


Figure 9.2: Use of the data structure to represent a Huffman tree.

We define $block_j$ as the block that groups all the nodes with j occurrences. We also define top_j and $last_j$ nodes respectively as the first and last nodes in the *node index* that are members of $block_j$.

The main advantage of maintaining a list of *blocks*, is that it can be easily maintained sorted in constant time. Assume that a word w_i, belonging to block b_j, is being processed. The number of occurrences of the leaf node q_i that represents w_i needs to be increased. That is, it is necessary to promote the node q_i from block b_j to block b_{j+1}, which basically implies two operations:

- Exchange q_i with top_j. Notice that top_j is the first node in the *node index* that belongs to the block b_j.
- Set $last_{j+1}$ to q_i and top_j to $q_i + 1$.

Figure 9.3 illustrates the process of changing a given node e from block b_4 to block b_5. In Figure 9.3(B) it is shown how e is exchanged with d, the *top* of its block b_4. Finally, e is set as the *last* node of b_5 in Figure 9.3(C).

When a node w_i is processed, several situations can take place depending on the state of the tree and the state of sorted list of blocks:

A. w_i was not in the tree: In this case, w_i is added to the tree, and set as the last node of b_1. See Figure 9.4(A).

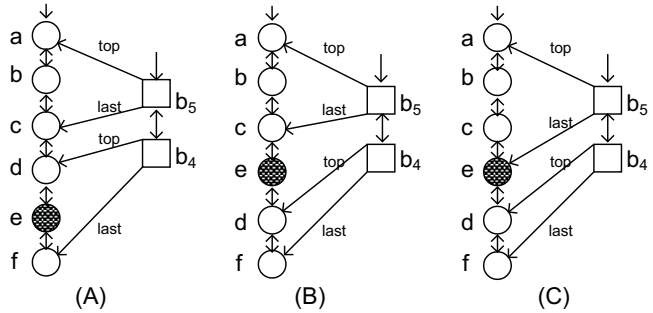


Figure 9.3: Increasing the frequency of word **e**.

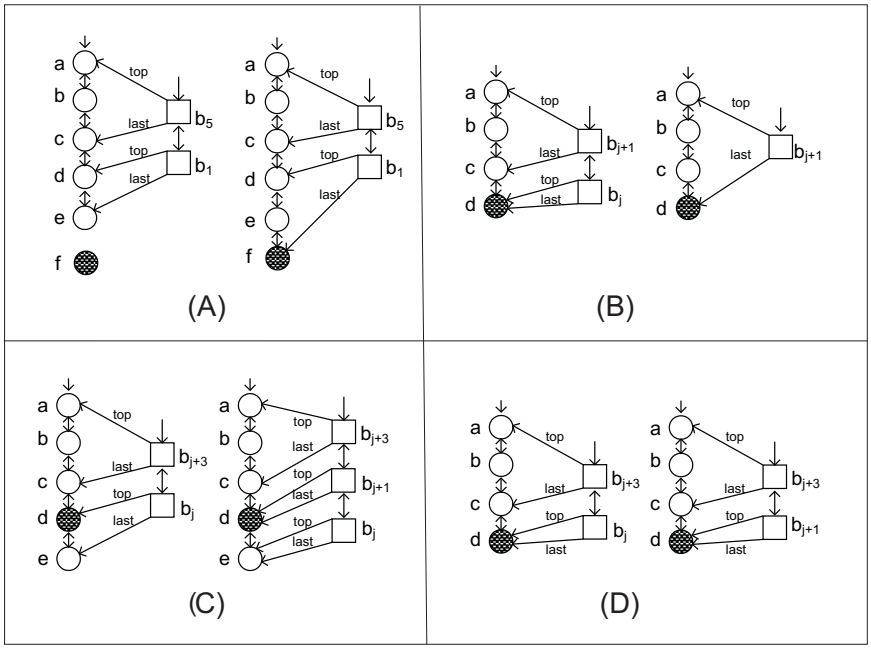


Figure 9.4: Distinct situations of increasing the frequency of a node.

B. w_i is the unique node in b_j and b_{j+1} exists. Since w_i is the only node in b_j, it is also the *top* of b_j. When w_i is promoted to block b_{j+1}, the block b_j becomes empty and has to be removed from the list of blocks. This scenario is shown in Figure 9.4(B).

C. w_i belongs to b_j and b_{j+1} does not exist. If there are two or more nodes in b_j then block b_{j+1} has to be added to the list of blocks (see Figure 9.4(C)), and then we operate as in case A.

D. w_i is the unique node of b_j and b_{j+1} does not exist. Therefore b_j is turned into block b_{j+1}, and neither insertions nor deletions of blocks are needed, as it is shown in Figure 9.4(D).

Implementation

Six vectors and a variable called *availableBlock* define the list of blocks data structure. The first vector is called *nodeInBlock* and stores information on nodes. The other five vectors hold information that is associated to blocks and permits maintaining a list of blocks. Those vectors are defined as follows:

- nodeInBlock[q] = i, $1 \leq q \leq N$, if and only if the node ranked q in the *node index* belongs to block i. That is, for each node in the *node index*, the vector *nodeInBlock* indicates in which block it appears.
- topBlock[i] = q, $1 \leq i \leq N$, if and only if the node ranked q in the *node index* is the *top* of block b_i.
- lastBlock[i] = q, $1 \leq i \leq N$, if and only if the node ranked q in the *node index* is the *last* node of block b_i.
- freq[i] = w, $1 \leq i \leq N$, if and only if the frequency of words in block b_i is w. Notice that now the frequency value of a node is not explicitly associated to it, since it can be obtained quickly. For example, the number of occurrences of a node q can be obtained as $freq[nodeInBlock[q]]$. As a result, the field $freq$ from the *node index* is no longer necessary.
- nextBlock[i] = nb, $1 \leq i \leq N$, if and only if block b_{nb} follows block b_i in the list of blocks. Since we are interested in maintaining also a list of unused blocks, this vector is initialized as:

$$nextBlock[i] \leftarrow i + 1;\ i \in 1..N - 1$$

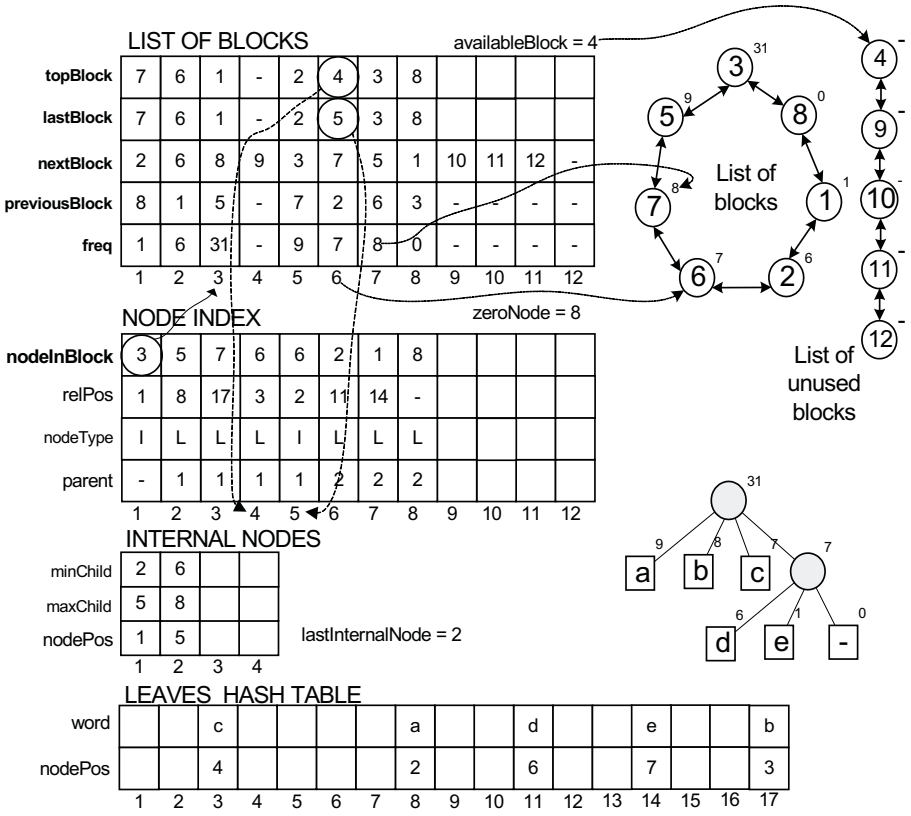


Figure 9.5: Huffman tree Data structure using a list of blocks.

- previousBlock[i] = nb, $1 \leq i \leq N$, if and only if block b_{nb} precedes b_i in the list of blocks. Notice that if $nextBlock[x] = y$ therefore it always holds that $prevBlock[y] = x$ (except for the unused blocks). That is, we are maintaining a circular list of used blocks.

- availableBlock indicates the first block that is unused. All unused blocks are linked together in a list using *nextBlock* array, and *availableBlock* is the header of that list. At the beginning of the process, *availableBlock* is initialized to 1.

The data structure presented in Figure 9.2 is integrated with the list of blocks as shown in Figure 9.5. It can be observed that each node (from the node index) is associated to one block through vector *nodeInBlock*. Moreover, it is immediate how to obtain both the *top* and the *last* node for a given block. Notice also that

a double-linked circular list of blocks is maintained through vectors *nextBlock* and *previousBlock*. Being double-linked it makes simple both deletions and insertions of blocks. Finally it is also shown that the unused blocks are also kept in a simple list, and that variable *availableBlock* always points to the first block in such list.

As additional implementation details, notice that, since it is necessary to reserve memory to hold the data structures used in our compressors and decompressors, we first obtained the size of the vocabulary (n) for each corpus. The experimental versions of our compressors and decompressors use n as a fixed parameter that permits to allocate enough memory to hold their data structures. However, the final versions of the programs should be able to use growing structures. For example, they can use Heaps' law to estimate an initial n from the size of the text to be compressed. Later, if data structures need to grow then the size of those structures can be duplicated (rehashing should be needed in the hash table) and then the compression/decompression process would continue. Using Heaps' law, the estimation $n = 4 \times (text\ size\ in\ bytes)^{0.6}$ became useful for all corpora in our experimental framework.

9.5 Huffman tree update algorithm

The compressor/sender and decompressor/receiver algorithms use the general guidelines shown in Figure 8.1.

The update algorithm is presented next. It assumes that q is the rank in the *node index* of the node whose frequency has to be increased (or $q = zeroNode$ if a new word is being added to the tree).

```
update(q)
(1)  findNode(q); //it sets q as the node to be increased
(2)  while q ≠ 1 do //bottom-up traversal of the tree
(3)       top ← topBlock[nodeInBlock[q]];
(4)       if q ≠ top then
(5)            if nodeType[q] =′ L′ then   //q is a leaf node
(6)                 if nodeType[top] =′ L′ then
(7)                      exchangeNodesLL (q, top);
(8)                 else  exchangeNodesIL (q, top);
(9)            else   //q is an internal node
(10)                if nodeType[top] =′ L′ then
(11)                     exchangeNodesIL (top, q);
(12)                else  exchangeNodesII (top, q);
```

(13) $q \leftarrow top$;
(14) increaseBlock(q); //it changes node q from block b_j to block b_{j+1}
(15) $freq[1] \leftarrow freq[1] + 1$; //root's frequency

How to exchange two nodes depends on their type. Next three algorithms enable exchanging either two leaves, two internal nodes or a leaf with an internal node.

exchangeNodesLL($leaf_i$, $leaf_j$)
(1) $lNodePos[relPos[leaf_i]] \leftrightarrow lNodePos[relPos[leaf_j]]$;
(2) $relPos[leaf_i] \leftrightarrow relPos[leaf_j]$;

exchangeNodesIL($leaf$, $internal$)
(1) $lNodePos[relPos[leaf]] \leftrightarrow iNodePos[relPos[internal]]$;
(2) $relPos[leaf] \leftrightarrow relPos[internal]$;
(3) $nodeType[leaf] \leftrightarrow nodeType[internal]$;

exchangeNodesII(i, j)
(1) $iNodePos[relPos[i]] \leftrightarrow iNodePos[relPos[j]]$;
(2) $relPos[i] \leftrightarrow relPos[j]$;

Procedure *findNode*() is shown next. Each time a new symbol s_i is transmitted (in this case $q = zeroNode$), a new node q_{s_i} needs to be added to the Huffman tree. If the node $pq = parent[q]$ has less than 2^b children (line 4), then it is enough to add q_{s_i} as a sibling of q. However, if the node $parent[q]$ has already 2^b children (line 37) then it is needed to first replace q by a new internal node npq (therefore npq is set as a child of pq), and later q_{s_i} and q are set as children of npq.

The algorithm *findNode()* deals with the operations needed to add such new symbol s_i to the tree. Notice that each call to *findNode()* sets the node q as the node that will be moved from block $b_j = nodeInBlock[q]$ to block b_{j+1} later.

findNode(q)
(1) $bq \leftarrow nodeInBlock[q]$;
(2) if $q = zeroNode$ then
(3) $pq \leftarrow parent[q]$;
(4) if $(maxChild[pq] - minChild[pq]) < 2^b - 1$ then //pq has less than 2^b

```
(5 )        nbq ← nextBlock[bq];      //children, so it has enough space to hold q.
(6 )        if freq[nbq] = 1 then //block "1" already exists
(7 )            nodeInBlock[q] ← 1;
(8 )            nodeInBlock[q + 1] ← 0;
(9 )            lastBlock[1] ← q;
(10)            lastBlock[0] ← q + 1;
(11)            topBlock[0] ← q + 1;
(12)        else //block "1" did not exists so it is created
(13)            bqI ← availableBlock;
(14)            availableBlock ← nextBlock[availableBlock];
(15)            nextBlock[bqI] ← nextBlock[bq];
(16)            previousBlock[bqI] ← previousBlock[nbq];
(17)            nextBlock[bq] ← bqI;
(18)            previousBlock[nbq] ← bqI;
(19)            freq[bqI] ← 1;
(20)            nodeInBlock[q] ← 1;
(21)            nodeInBlock[q + 1] ← 0;
(22)            topBlock[bqI] ← q;
(23)            lastBlock[bqI] ← q;
(24)            topBlock[bq] ← q + 1;
(25)            lastBlock[bq] ← q + 1;
(26)        //q is added to the tree
(27)        addr ← f_hash(s_i);
(28)        word[addr] ← s_i;
(29)        lNodePos[addr] ← q;
(30)        relPos[q] ← addr;
(31)        nodeType[q] ← 'L';
(32)        parent[q] ← pq;
(33)        zeroNode ← zeroNode + 1;
(34)        nodeType[zeroNode] ← 'L';
(35)        parent[zeroNode] ← pq;
(36)        maxChild[pq] ← maxChild[pq] + 1;
(37)    else //a new internal node has to be created in position zeroNode
(38)        lastIntNode ← lastIntNode + 1;
(39)        npq ← lastIntNode;
(40)        relPos[q] ← npq;
(41)        nodeType[q] ← 'I';
(42)        parent[q] ← pq;
(43)        minChild[npq] ← q + 1;
(44)        maxChild[npq] ← q + 2;
(45)        iNodePos[npq] ← q;
(46)        nbq ← nextBlock[bq];
(47)        if freq[nbq] = 1 then //block "1" already exists
(48)            nodeInBlock[q] ← 1;
(49)            nodeInBlock[q + 1] ← 1;
(50)            nodeInBlock[q + 2] ← 0;
(51)            lastBlock[1] ← q + 1;
(52)            lastBlock[0] ← q + 2;
(53)            topBlock[0] ← q + 2;
(54)        else //block "1" does not exist yet
(55)            bqI ← availableBlock;
```

```
(56)                availableBlock ← nextBlock[availableBlock];
(57)                nextBlock[bqI] ← nextBlock[bq];
(58)                previousBlock[bqI] ← previousBlock[nbq];
(59)                nextBlock[bq] ← bqI;
(60)                previousBlock[nbq] ← bqI;
(61)                freq[bqI] ← 1;
(62)                nodeInBlock[q] ← 1;
(63)                nodeInBlock[q + 1] ← 1;
(64)                nodeInBlock[q + 2] ← 0;
(65)                topBlock[bqI] ← q;
(66)                lastBlock[bqI] ← q + 1;
(67)                topBlock[bq] ← q + 2;
(68)                lastBlock[bq] ← q + 2;
(69)            q ← q + 1;
(70)            addr ← f_hash(currentWord);
(71)            word[addr] ← currentWord;
(72)            lNodePos[addr] ← q;
(73)            relPos[q] ← addr;
(74)            nodeType[q] ← 'L';
(75)            parent[q] ← npq;
(76)            zeroNode ← zeroNode + 2;
(77)            nodeType[zeroNode] ← 'L';
(78)            parent[zeroNode] ← npq;
(79)        q ← intNodes[pq].nodePos;
(80) else //q is not the zeroNode, so it is already the node to be increased
```

Note that the function $f_{hash}(currentWord)$ used in lines 27 and 70 returns the position inside the hash table where $currentWord$ should be placed. Note also that in the case of the decompressor, as it needs only an array of words, rather than a hash table, these lines should be replaced by next two instructions:

$$addr \leftarrow nextFreeWord;\ nextFreeWord \leftarrow nextFreeWord + 1;$$

where $nextFreeWord$ stores the next available free position in *words* vector.

The last procedure used in *update*() is called *increaseBlock*(q). This procedure is called in each step of the bottom-up traversal of the Huffman tree. It increases the frequency of the node q and sets q to point to its parent, next level upward in the tree.

9.6 Empirical results

We compressed the texts shown in Section 2.7 to test the compression ratio and time performance of the character-based *FGK algorithm* (using the *compact*

```
increaseBlock(q)
(1 ) bq ← nodeInBlock[q];
(2 ) nbq ← nextBlock[bq];
(3 ) if freq[nbq] = (freq[bq] + 1) then
(4 )       nodeInBlock[q] ← nbq;
(5 )       lastBlock[nbq] ← q;
(6 )       if lastBlock[bq] = q then //q's old block disappears
(7 )             previousBlock[nbq] ← previousBlock[bq];
(8 )             nextBlock[previousBlock[bq]] ← nbq;
(9 )             nextBlock[bq] ← availableBlock;
(10)             availableBlock ← bq;
(11)       else topBlock[bq] ← q + 1;
(12) else //Next block does not exist
(13)       if lastBlock[bq] = q then
(14)             freq[bq] ← freq[bq] + 1;
(15)       else
(16)             b ← availableBlock;
(17)             availableBlock ← nextBlock[availableBlock];
(18)             nextBlock[bq] ← b;
(19)             previousBlock[b] ← bq;
(20)             nextBlock[b] ← nbq;
(21)             previousBlock[nbq] ← b;
(22)             topBlock[bq] ← q + 1;
(23)             topBlock[b] ← q;
(24)             lastBlock[b] ← q;
(25)             freq[b] ← freq[bq] + 1;
(26)             nodeInBlock[q] ← b;
(27) q ← iNodePos[parent[q]];
```

command) against the dynamic word-based byte-oriented Huffman code described in this chapter. That comparison is shown in Section 9.6.1. In Section 9.6.2, we also compare our dynamic word-based byte-oriented Huffman code against Plain Huffman (semi-static code) in compression ratio, and compression and decompression speed.

9.6.1 Character- versus word-oriented Huffman

Table 9.1 compares the compression ratio and compression speed of both techniques. As expected, the word-based technique reduces the compression ratio of the character-based dynamic Huffman by half. Compression speed is also increased. The dynamic word-based byte-oriented Huffman (DynPH) reduces *FGK algorithm* compression time in about 8.5 times. Note that, for using bytes instead of bits

and for improving the compression ratio by half, the compression time should have been reduced up to 16 times, but it was reduced only by 8.5 times. This happened because a 46% overhead arises as the result of using more complex algorithms.

CORPUS	O SIZE	Dyn PH			*FGK* algorithm		
	bytes	bytes	ratio	time (sec)	bytes	ratio	time(sec)
CALGARY	2,131,045	991,911	46.546	0.342	1,315,774	61.743	3.270
FT91	14,749,355	5,123,739	34.739	2.322	9,109,215	61.760	20.020
CR	51,085,545	15,888,830	31.102	7.506	31,398,726	61.463	71.010
FT92	175,449,235	56,185,629	32.024	29.214	108,420,660	61.796	259.125
ZIFF	185,220,215	60,928,765	32.895	30.218	117,703,299	63.548	264.570
FT93	197,586,294	63,238,059	32.005	32.506	124,501,969	63.011	272.375
FT94	203,783,923	65,126,566	31.959	33.310	128,274,442	62.946	278.210
AP	250,714,271	80,964,800	32.294	43.228	155,010,899	61.828	361.120
ALL_FT	591,568,807	187,586,995	31.710	103.354	370,551,380	62.639	832.134
ALL	1,080,719,883	355,005,679	32.849	209.476	678,287,443	62.763	1,523.890

Table 9.1: Word-based Vs character-based dynamic approaches.

9.6.2 Semi-static Vs dynamic approach

Table 9.2 compares the compression ratio of Plain Huffman (semi-static version) with that of the dynamic version presented in this chapter. The first four columns in that table indicate respectively the corpus, the size (in bytes), the size of the vocabulary, and the number of words in the whole text. The fifth and sixth columns show the compression ratio obtained by the semi-static and dynamic techniques respectively. Finally, the last column shows the gain (in percentage) of the semi-static code with respect to the dynamic one.

CORPUS	TEXT SIZE	n	#words	semi-static	dynamic	diff
	bytes			ratio%	ratio%	ratio%
CALGARY	2,131,045	30,995	528,611	46.238	46.546	0.308
FT91	14,749,355	75,681	3,135,383	34.628	34.739	0.111
CR	51,085,545	117,713	10,230,907	31.057	31.102	0.046
FT92	175,449,235	284,892	36,803,204	32.000	32.024	0.024
ZIFF	185,220,215	237,622	40,866,492	32.876	32.895	0.019
FT93	197,586,294	291,427	42,063,804	31.983	32.005	0.022
FT94	203,783,923	295,018	43,335,126	31.937	31.959	0.022
AP	250,714,271	269,141	53,349,620	32.272	32.294	0.021
ALL_FT	591,568,807	577,352	124,971,944	31.696	31.710	0.014
ALL	1,080,719,883	886,190	229,596,845	32.830	32.849	0.019

Table 9.2: Compression ratio of dynamic versus semi-static versions.

It can be seen that adding dynamism to Plain Huffman produces a negligible loss of compression ratio (under 0.05 percentage points in general).

We also measured the compression and decompression speed of our dynamic version to compare it with Plain Huffman. Table 9.3 shows the results obtained. The first column indicates the corpus processed, and the second, the number of words in the source text. Observe that the number of words in the source text (or

the number of codewords in a compressed text) corresponds exactly with the number of times that the update algorithm is called during compression and decompression processes. Columns three and four in Table 9.3 show, respectively, the time (in seconds) needed to run the semi-static and the dynamic compressor over each corpus. The gain of compression speed (in percentage) obtained by the semi-static compressor with respect to the dynamic one, is given in the fifth column. The sixth and seventh columns in that table show decompression time (in seconds), and the last one shows the gain in decompression speed of Plain Huffman against the dynamic version of the code.

CORPUS	#words	Compression time (sec)			Decompression time (sec)		
		semi-static	dynamic	diff %	semi-static	dynamic	diff %
CALGARY	528,611	0.415	0.342	-21.345	0.088	0.216	59.105
FT91	3,135,383	2.500	2.322	-7.666	0.577	1.554	62.891
CR	10,230,907	7.990	7.506	-6.448	1.903	5.376	64.611
FT92	36,803,204	29.243	29.214	-0.098	7.773	21.726	64.225
ZIFF	40,866,492	30.354	30.218	-0.451	8.263	21.592	61.730
FT93	42,063,804	32.915	32.506	-1.258	8.406	23.398	64.075
FT94	43,335,126	33.874	33.310	-1.692	8.636	23.802	63.716
AP	53,349,620	42.641	43.228	1.357	11.040	32.184	65.697
ALL_FT	124,971,944	99.889	103.354	3.353	24.798	75.934	67.343
ALL	229,596,845	191.396	209.476	8.631	45.699	161.448	71.695

Table 9.3: Compression and decompression speed comparison.

As shown in Section 6.6.2, the semi-static technique processes twice each corpus during compression. After the first pass each word in the vocabulary is given a codeword. That association remains fixed during the second pass, where the compressor has only to substitute each source word by its corresponding symbol. In the dynamic compressor, the source text is processed only once. However, the association word-codeword (given by the Huffman tree) varies as the compression progresses. Each time a source word is processed, it has to be encoded using the current Huffman tree. This tree is then updated to maintain it optimal. As a result, both the encoding and the update algorithms are performed as many times as words exist in the source text.

In small corpora, the dynamic compressor takes advantage of performing a unique pass over the text. However, as the size of the corpora increases, the cost of performing two passes is compensated by the large number of times that the encoding and the update algorithms are called in the dynamic compressor. This happens mainly because calls to the update algorithm become more and more expensive (the height of the Huffman tree increases) as the vocabulary grows.

Regarding decompression, the semi-static technique is much simpler. It loads the whole vocabulary first (and the shape of the Huffman tree used in compression) and then it has only to decode each codeword to obtain its corresponding uncompressed word. On the other hand, the dynamic decompressor starts with an empty Huffman

tree, that must be updated each time a codeword is decoded. Since updating the dynamic Huffman tree is a complex task, the decompression speed obtained by the semi-static code overcomes that of the dynamic decompressor by up to 71%. (It becomes almost 4 times faster than the dynamic one).

More empirical results comparing the Dynamic word-based byte-oriented Huffman technique explained in this chapter against several well-known compression techniques, are also presented in Section 11.5.

9.7 Summary

Our dynamic word-based byte-oriented Huffman code and the data structures that support efficiently the update process of the Huffman tree in each step of both compression and decompression processes were presented.

Word-based Huffman codes are known to obtain good compression. For this sake, we adapted an existing character-based Huffman algorithm to obtain a word-based one. Our new dynamic Huffman code can handle very large sets of source words and produces a byte-oriented output. The latter decision sacrifices some compression ratio in exchange for an 8-fold improvement in time performance. The resulting algorithm obtains a compression ratio very similar to its semi-static version (only 0.05 percentage points off).

Regarding compression speed, the new dynamic technique is faster than Plain Huffman in small- and medium-sized texts. However, compression speed worsens as the size of the text collection increases. In the case of decompression speed, the dynamic decompressor obtains acceptable results, but, as expected, it is much slower than Plain Huffman.

Some empirical results comparing the new technique with the *FGK algorithm*, a good character-based dynamic Huffman method, were presented. Those results show that our implementation clearly improves the compression ratio and largely reduces the compression time achieved by the character-based *FGK* algorithm.

As a result, we have obtained an adaptive natural language text compressor that obtains 30%-32% compression ratio for large texts, and compresses more than 5 Mbytes per second on a standard PC. Empirical results also show its good performance when compared against other compressors such as *gzip*, *bzip2* and arithmetic encoding (see Section 11.5 for more details).

Having set the efficiency of this algorithm, we will use it as a competitive

compression technique to show that the dynamic versions of both End-Tagged Dense Code and (s, c)-Dense Code are, as their *two-pass* versions, good alternatives to Huffman compression techniques, because of their competitive compression ratio, compression and decompression speed, and also, due to the easiness of their implementation.

10

Dynamic End-Tagged Dense Code

This chapter presents a dynamic version of the End-Tagged Dense Code. First, the motivation of this new technique is explained. Then, its basis are presented in Section 10.2. The data structures needed to make it an efficient adaptive technique, are explained in Section 10.3. Section 10.4 presents pseudo-code for both compressor and decompressor processes. It includes the algorithms to adapt the model used when a source symbol is processed. In Section 10.5, some empirical results are given. We first compare both the dynamic and the semi-static versions of End-Tagged Dense Code in compression ratio and time performance. Then, we also compare the new dynamic code with the Dynamic Huffman code shown in the previous chapter. Finally, some conclusions regarding to this new technique are given.

10.1 Motivation

In Chapter 9, a good word-based byte-oriented dynamic Huffman code was presented. That code joins the good compression ratios achieved by the word-based semi-static statistical Huffman methods (in this case, Plain Huffman method [MNZBY00]) with the advantages of adaptive compression techniques in file transmission scenarios. The resulting code permits real-time transmission of data and achieves compression ratios very close to those of the semi-static version of the

code.

However, this algorithm is rather complex to implement and the update of the Huffman tree is time-consuming. Each time a symbol s_i is processed, it is required to perform a full bottom-up traversal of the tree to update the weights of all the ancestors of s_i until reaching the *root* (and at each level, reorganizations can happen).

In Chapter 5, we described End-Tagged Dense Code, a statistical word-based compression technique (not using Huffman). End-Tagged Dense Code was shown to be a good alternative to word-based Huffman due to several features:

- It is very simple to build since only a list of words ranked by frequency is needed by both the encoding and decoding processes.

- End-Tagged Dense Code is faster than Huffman-based codes. Code generation process is about $50 - 60\%$ faster than in the case of Plain Huffman, while compression and decompression speed are similar in both cases.

- The loss of compression ratio compared against the Plain Huffman code is small (about 1 percentage point).

- It enables to use direct searches inside the compressed text.

For the first three reasons it is interesting to develop a Dynamic End-Tagged Dense Code. This new code [BFNP04] is shown in the next sections of this chapter. It improves the compression/decompression speed of the dynamic Huffman-based technique (about 20%-24% faster in compression and 45% in decompression), at the expense of a small loss of compression ratio (one percentage point).

It is interesting to point out that the semi-static End-Tagged Dense Code is much faster than Plain Huffman in code generation phase (about 60%), but since this phase is a very small part of the overall two-pass compression process, differences in compression time between both techniques do not exist. However, when dynamic techniques are considered, encoding is a very significative part of the overall process, hence differences in time between Dynamic End-Tagged Dense Code and Dynamic Huffman are substantial.

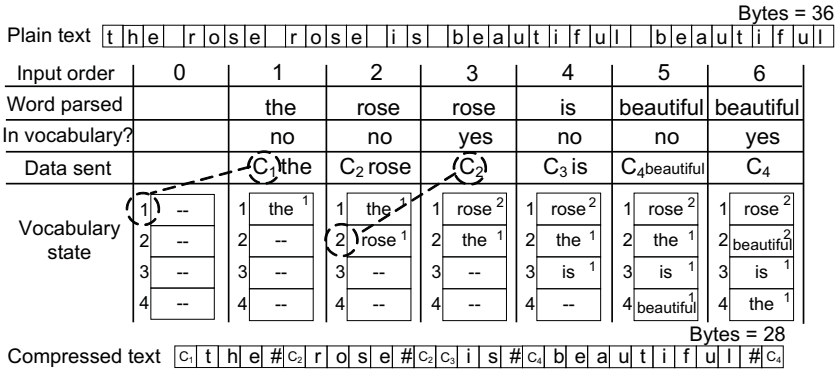


Figure 10.1: Transmission of `"the rose rose is beautiful beautiful"`.

10.2 Method overview

In this Section it is shown how ETDC can be made dynamic. Considering again the general scheme of Figure 8.1, the main issue is how to maintain the *CodeBook* up to date upon insertions of new source symbols and frequency increments.

In the case of ETDC, the *CodeBook* consists essentially of one structure that keeps the vocabulary ordered by frequency. Since we maintain such sorted vocabulary upon insertions and frequency changes, we can encode any source symbol or decode any target symbol by using the on-the-fly *encode* and *decode* procedures explained in Section 5.3.

Figure 10.1 shows how the compressor operates. At first (step 0), no words have been read so *new-Symbol* is the only word in the vocabulary (it is implicitly placed at position 1). In step 1, a new symbol `"the"` is read. Since it is not in the vocabulary, C_1 (the codeword of *new-Symbol*) is sent, followed by `"the"` in plain form (bytes `'t'`, `'h'`, `'e'` and some terminator `'#'`). Then `"the"` is added to the vocabulary with frequency 1, at position 1. Implicitly, *new-Symbol* has been displaced to position 2. Step 2 shows the transmission of `"rose"`, which was not yet in the vocabulary. In step 3, `"rose"` is read again. Since it was in the vocabulary at position 2, only the codeword C_2 is sent. Now, `"rose"` becomes more frequent than `"the"`, so it moves upwards in the ordered vocabulary. Note that a hypothetical new occurrence of `"rose"` would be transmitted as C_1, while it was sent as C_2 in step 3. In steps 4 and 5, two more new words, `"is"` and `"beautiful"`, are transmitted and added to the vocabulary. Finally, in step 6, `"beautiful"` is read again, and when its frequency is updated it becomes more frequent than `"is"` and `"the"`. Therefore,

it moves upwards in the vocabulary by means of an exchange with `"the"` (which is the first word in its block of frequencies).

The receiver works similarly to the sender. It receives a codeword C_i, and decodes it. As a result of decoding C_i, a symbol S_i is obtained. If C_i corresponds to the *new-Symbol*, then the receiver knows that a new word s_i will be received in plain form next, so s_i is received and added to the vocabulary. When C_i corresponds to a word s_i that is already in the vocabulary, the receiver only has to increase its frequency and usually to reorder the vocabulary. Reordering the vocabulary only implies (when needed), to exchange the word s_i with the first word with the same frequency of s_i.

The main challenge is how to efficiently maintain the vocabulary sorted. In Section 10.3 it is shown how to do this with a complexity equal to the number of source symbols transmitted. This is always lower than *FGK* complexity, because at least one target symbol must be transmitted for each source symbol, and usually several more.

Essentially, we must be able to identify *blocks* of words with the same frequency in the ordered vocabulary, and be able to fast promote a word to the next block when its frequency increases. Promoting a word w_i with frequency f to next frequency blocks $f+1$ consists of:

- Sliding w_i over all words whose frequency is f. This operation implies two operations:
  - Locating the first ranked word in the ordered vocabulary whose frequency is f. This word is called top_f.
  - Exchanging w_i with top_f
- Increasing the frequency of w_i

The data structures and the way they are used to efficiently maintain the vocabulary sorted are shown next.

10.3 Data structures

As with word-based dynamic Huffman (Chapter 9), the sender maintains a hash table that permits fast searching for a source word s_i. The hash table is also used to obtain the rank i in the vocabulary vector (remember that to encode a word

s_i only its rank i is needed), as well as its current frequency f_i (which is used to rapidly find the position of the word top_{f_i}).

The receiver does not need to maintain a hash table to hold words. It only needs to use a *word* vector, because the decoding process uses the codeword to directly obtain the rank value i that can be used to index the *word* vector. Finding a word lexicographically is not necessary.

Let n be the vocabulary size, F the maximum frequency value for any word in the vocabulary (it is estimated heuristically). The data structures used by both the sender and the receiver, as well as their functionality, are shown next.

10.3.1 Sender's data structures

The following three main data structures, shown in Figure 10.2, are needed:

- A *hash table* of words keeps in *word* the source word, in *posInVoc* the rank (or position) of the word in the ordered vocabulary, and in *freq* its frequency. Both *word*, *posInVoc*, and *freq* are H-element arrays (having $H = nextPrime(2n)$).
- In the *posInHT* array, each position represents a specific word of the vocabulary. Words are not explicitly represented, but a pointer to the *word* vector in the hash table is stored. *posInHT* is an n-element vector. This vector keeps words ordered by frequency. That is, $posInHT[1]$ points to the most frequent word, $posInHT[2]$ to the second most frequent word, and so on.
- Array *top* is defined as an F-element array. Each position represents a frequency value implicitly. That is, *top*[1] is associated to words with frequency equal to 1, *top*[2] represents words with frequency 2, an so on. For each possible frequency, *top* vector keeps a pointer to the entry in *posInHT* that represent the rank of the first word with that frequency.

Two more variables, *new-Symbol* and *maxFreq*, are needed. *new-Symbol* holds the first free position in the vocabulary (in *posInHT*) and *maxFreq* the first free position that appears at the end of the *top* vector.

Using the F-element vector *top* may require a large amount of memory. However, since *top* maintains an implicit *list of blocks* structure, it results faster and simpler than maintaining an explicit *list of blocks* as in the dynamic Huffman technique presented in Chapter 9. Besides, if memory requirements need to be reduced, it is

also possible to substitute this F-element *top* vector by a set of 6 vectors of size n, to maintain an explicit *list of blocks* in the same way that it was presented in Section 9.4.2.

The highest frequency value, which was obtained in our experiments with the largest corpus (ALL), was $F = 8,205,778$. In this corpus, the number of words in the vocabulary was $n = 885,873$. Therefore the space requirements to keep vector *top* is $8,205,778 \times 4$ bytes $= 32,823,112$ bytes ≈ 31 Mbytes, while using an explicit *list of blocks* need $6 \times 885,873 \times 4$ bytes $= 21,260,852$ bytes ≈ 20 Mbytes. Both are comparable and perfectly reasonable for current computers.

Notice that using the slower approach (the explicit list of blocks) is mandatory in the case of our dynamic Huffman technique, since in this case F would correspond to the *root frequency*, that is, the number of words in the source text. Therefore the amount of memory required to keep the *top* vector would be too large. For example, using the faster approach (a unique *top* vector with F elements) in the ALL corpus, which has $229,596,845$ words, would need $229,596,845 \times 4 = 918,387,380$ bytes ≈ 876 Mbytes!!

10.3.2 Receiver's data structures

The following three vectors are needed:

- A *word* vector that keeps the source words. Its size is n.
- A *freq* vector that keeps the frequency of each word. That is, $freq[i] = f$, if the number of occurrences of the word stored in $word[i]$ is f. As the *word* array, this vector can keep up to n elements.
- Array *top*. As it happened in the sender, this array gives, for each possible frequency, the word position of the first word with that frequency. It has also F positions.

Variables *new-Symbol* and *maxFreq* are also needed by the receiver. All the structures needed by the receiver are represented in Figure 10.3.

Next section explains the way both sender and receiver work, and how they use the data structures presented to make sending/compression and reception/decompression processes efficient.

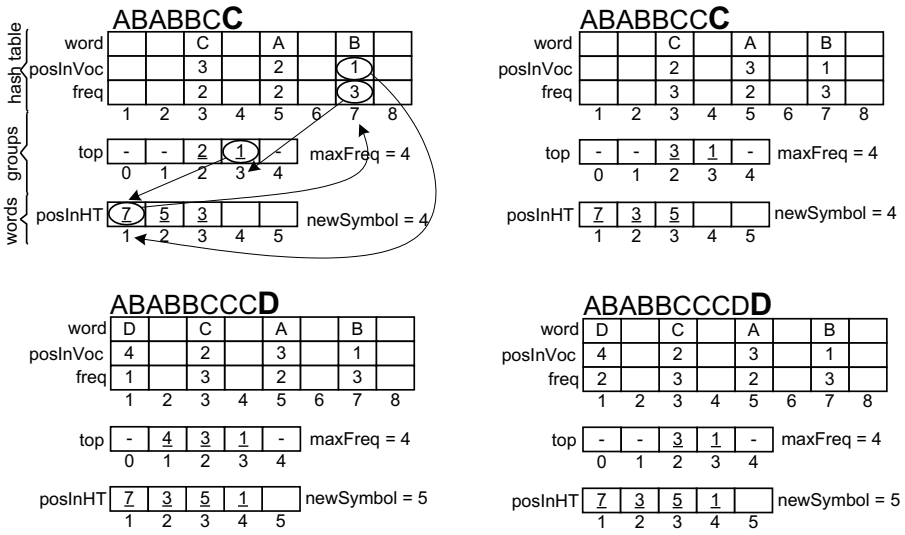


Figure 10.2: Transmission of words C, C, D and D having transmitted ABABB earlier.

10.4 Sender's and receiver's pseudo-code

When the sender reads word s_i, it uses the hash function to obtain its position p in the hash table, so that $f_{hash}(s_i) = p$ and therefore $word[p] = s_i$. After reading $f = freq[p]$, it increments $freq[p]$. The position of s_i in the vocabulary array is also obtained as $i = posInVoc[p]$ (the codeword C_i can be computed and sent using the on-the-fly algorithm shown in page 73). Now, word s_i must be promoted to the next block. For this sake, sender's algorithm finds the head of its block $j = top[f]$ and the corresponding position h of the word in the hash table $h = posInHT[j]$. Now, it is necessary to swap words i and j in the vector $posInHT$. The swapping requires exchanging $posInHT[j] = h$ with $posInHT[i] = p$, setting $posInVoc[p] = j$ and setting $posInVoc[h] = i$. Once the swapping is done, we promote j to the next block by setting $top[f] = j + 1$.

If s_i turns out to be a new word, we set $word[p] = s_i$, $freq[p] = 0$, and $posInVoc[p]$ =*new-Symbol*. Then exactly the above procedure is followed with $f = 0$ and i =*new-Symbol*. Finally *new-Symbol* is also increased.

Figure 10.2 explains the way the sender works and how its data structures are used.

The receiver works very similarly to the sender, but it is even simpler. Given

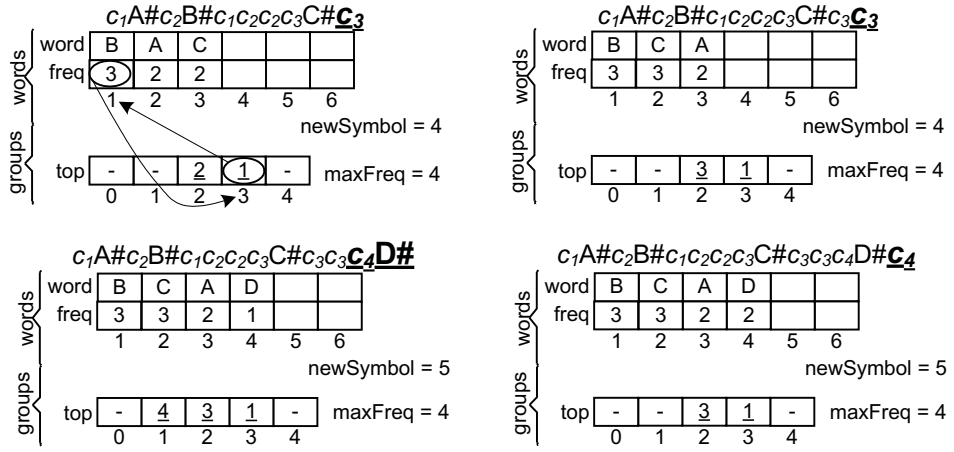


Figure 10.3: Reception of c_3, c_3, $c_4D\#$ and c_4 having previously received c_1A#c_2B#$c_1c_2c_2c_3$C#.

a codeword C_i, the receiver decodes it using the decode algorithm in page 74, obtaining the position i such as decode$(C_i) = i$ and $word[i]$ contains the word s_i that corresponds to C_i. Therefore $word[i]$ can be output. Next the receiver sets $f = freq[i]$ and then increases $freq[i]$. In order to promote s_i in the vocabulary, $j = top[f]$ is located. In next step $word[i]$ and $word[j]$, as well as $freq[i]$ and $freq[j]$, are swapped. Finally j is promoted to the group of frequency $f + 1$ by setting $top[f] = j + 1$.

If $i = new\text{-}Symbol$ then a new word s_i is being transmitted in plain form. We set $word[i] = s_i$, $freq[i] = 0$, and again the previous process is performed with $f = 0$ and $i =$*new-Symbol*. Finally *new-Symbol* is also increased.

Figure 10.3 gives an example of how the receiver works.

Pseudo-code for both sender and receiver processes is shown in Figures 10.4 and 10.5. Notice that implementing Dynamic ETDC is simpler than building dynamic word-based Huffman. In fact, our implementation of the Huffman tree update (Section 9.5) takes about 120 C source code lines, while the update procedure takes less than 20 lines in Dynamic ETDC.

10.5 Empirical results

We compressed the real texts described in Section 2.7 to test the compression ratio of the one- and two-pass versions of End-Tagged Dense Code (ETDC) and Plain Huffman (PH). We also compared compression and decompression speed for both the semi-static and the dynamic versions of ETDC. These results are shown in Section 10.5.1. Finally, a comparison between the compression and decompression speed obtained by the dynamic versions of ETDC and Plain Huffman is given in Section 10.5.2.

10.5.1 Semi-static Vs dynamic approach

Table 10.1 compares the compression ratio of two-pass versus one-pass techniques. Columns labelled **diff** measure the increase (in percentage points) in the compression ratio of the dynamic codes with respect to their semi-static version. The last column compares those differences between Plain Huffman and ETDC.

CORPUS	TEXT SIZE	Plain Huffman			End-Tagged Dense Code			diff
		2-pass	1-pass	diff	2-pass	1-pass	diff	PH-
	bytes	ratio%	ratio%	PH	ratio%	ratio%	ETDC	ETDC
CALGARY	2,131,045	46.238	46.546	0.308	47.397	47.730	0.332	-0.024
FT91	14,749,355	34.628	34.739	0.111	35.521	35.638	0.116	-0.005
CR	51,085,545	31.057	31.102	0.046	31.941	31.985	0.045	0.001
FT92	175,449,235	32.000	32.024	0.024	32.815	32.838	0.023	0.001
ZIFF	185,220,215	32.876	32.895	0.019	33.770	33.787	0.017	0.002
FT93	197,586,294	31.983	32.005	0.022	32.866	32.887	0.021	0.001
FT94	203,783,923	31.937	31.959	0.022	32.825	32.845	0.020	0.002
AP	250,714,271	32.272	32.294	0.021	33.087	33.106	0.018	0.003
ALL_FT	591,568,807	31.696	31.710	0.014	32.527	32.537	0.011	0.003
ALL	1,080,719,883	32.830	32.849	0.019	33.656	33.664	0.008	0.011

Table 10.1: Compression ratios of dynamic versus semi-static techniques.

Compression ratios are around 31-34% in the largest texts. In the smallest, compression is poor because the size of the vocabulary is proportionally too large with respect to the compressed text size.

From the experimental results, it can also be seen that the cost of dynamism in terms of compression ratio is negligible. The dynamic versions lose very little in compression (around 0.02 percentage points) compared to their semi-static versions. Moreover, in most texts (the positive values in the last column) Dynamic ETDC loses even less compression than the Dynamic Plain Huffman.

In Table 10.2, we show the influence of dynamism in ETDC in terms of compression and decompression speed. Columns three and four give compression time and columns six and seven give decompression time for both ETDC and

CORPUS	TEXT SIZE	Compression time (sec)			Decompression time (sec)		
	bytes	2-pass	1-pass	diff (%)	2-pass	1-pass	diff (%)
CALGARY	2,131,045	0.393	0.268	46.584	0.085	0.122	-30.137
FT91	14,749,355	2.482	1.788	38.770	0.570	0.847	-32.677
CR	51,085,545	7.988	6.050	32.029	1.926	3.033	-36.520
FT92	175,449,235	29.230	22.905	27.614	7.561	11.603	-34.837
ZIFF	185,220,215	30.368	24.042	26.312	7.953	11.888	-33.104
FT93	197,586,294	32.783	25.505	28.535	8.694	12.675	-31.410
FT94	203,783,923	33.763	26.385	27.961	8.463	13.013	-34.971
AP	250,714,271	42.357	33.700	25.689	11.233	16.923	-33.622
ALL_FT	591,568,807	100.469	79.307	26.684	24.500	39.970	-38.704
ALL	1,080,719,883	191.763	157.277	21.927	46.352	81.893	-43.400

Table 10.2: Comparison in speed of ETDC and Dynamic ETDC.

Dynamic ETDC respectively (in seconds). The fifth column shows the increment of compression time (in percentage) obtained by the semi-static code with respect to the Dynamic ETDC. Finally, the last column in that table presents (in percentage) the difference in decompression time between ETDC and the dynamic method, showing that ETDC is faster.

In all the corpora used, the compression speed obtained by the Dynamic ETDC is much better (at least 21%) than that of the 2-pass approach. As it happened in the previous chapter, differences between both approaches decrease as the size of the source texts grows. However, the update algorithm is now so simple and fast that those distances in time decrease with a very gentle slope. This becomes an important advantage over the dynamic Huffman technique presented in the previous chapter. In that case, as the size of the corpora increased, differences between Plain Huffman and the dynamic Huffman-based technique decreased so fast that, in the two largest corpora, the dynamic compressor was overcome by the semi-static compressor. In decompression speed, the semi-static ETDC clearly overcomes the results obtained by the dynamic technique. Indeed, it reduces the time needed in decompression by the Dynamic ETDC in more than 31%.

10.5.2 Dynamic ETDC Vs dynamic Huffman

As it happened in the semi-static versions of the two codes, compression ratio is around 30%-35% in both Dynamic ETDC and Dynamic PH. As expected, the compression ratio obtained by Dynamic ETDC is around 1 percentage point worse than that of Dynamic PH. Those compression ratios are shown in Table 10.1.

Compression and decompression time for both Dynamic ETDC and Dynamic PH are compared in Table 10.3. Columns labelled **diff** show (in percentage) the advantage in time of Dynamic ETDC with respect to Dynamic PH. In compression, Dynamic ETDC is 20%-25% faster than Dynamic PH. If we consider decompression, the simpler decompression mechanism of Dynamic ETDC makes it much faster than

CORPUS	Compression time (sec)			Decompression time (sec)		
	Dyn PH	DETDC	diff (%)	Dyn PH	DETDC	diff(%)
CALGARY	0.342	0.268	21.540	0.216	0.122	43.673
FT91	2.322	1.788	22.983	1.554	0.847	45.517
CR	7.506	6.050	19.398	5.376	3.033	43.576
FT92	29.214	22.905	21.596	21.726	11.603	46.592
ZIFF	30.218	24.042	20.439	21.592	11.888	44.941
FT93	32.506	25.505	21.538	23.398	12.675	45.829
FT94	33.310	26.385	20.790	23.802	13.013	45.327
AP	43.228	33.700	22.041	32.184	16.923	47.417
ALL_FT	103.354	79.307	23.267	75.934	39.970	47.362
ALL	209.476	157.277	24.919	161.448	81.893	49.276

Table 10.3: Comparison of compression and decompression time.

Dynamic PH. As a result, an improvement of about 45% is achieved.

More empirical results, comparing the *one-pass* version of ETDC with other well-known compression techniques, are shown in Section 11.5.

10.6 Summary

We adapted End-Tagged Dense Code (ETDC) to obtain a dynamic version of that compressor. The resulting dynamic version is much simpler than the Huffman-based one (proposed in Chapter 9). This is because maintaining an ordered list of words is much simpler than adapting a Huffman tree (and less operations are performed). As a result, Dynamic ETDC is 20%-25% faster than Dynamic PH in compression, compressing typically 7 Mbytes per second. In decompression, Dynamic ETDC is about 45% faster than Dynamic PH.

Comparing Dynamic ETDC with the semi-static version, the compression ratio is only 0.02% larger than with semi-static ETDC and 1% larger than with Huffman. In compression speed, Dynamic ETDC is faster (more than 20%) than the semi-static version of ETDC, but it is slower in decompression (around 30%-35%).

```
Sender main algorithm ( )
(1)  new-Symbol ← 1;
(2)  top[0] ← new-Symbol;
(3)  maxFreq ← 1;
(4)  while more words left
(5)      read s_i from text;
(6)      if (s_i ∉ word) then
(7)          p ← f_hash(s_i);
(8)          i ← new-Symbol;
(9)          send (encode(i));
(10)         send s_i in plain form;
(11)     else
(12)         i ← posInVoc[p];
(13)         send (encode(i));
(14)     update();
```

```
Sender update ( )
(1)  if i =new-Symbol then // new word
(2)      word[p] ← s_i;
(3)      freq[p] ← 0;
(4)      posInVoc[p] ← new-Symbol ;
(5)      posInHT[new-Symbol] ← p;
(6)      new-Symbol ← new-Symbol + 1;
(7)  f ← freq[p];
(8)  freq[p] ← freq[p] + 1;
(9)  j ← top[f];
(10) h ← posInHT[j];
(11) posInHT[i] ↔ posInHT[j];
(12) posInVoc[p] ← j;
(13) posInVoc[h] ← i;
(14) top[f] ← j + 1;
(15) if maxFreq = f + 1 then
(16)     top[f + 1] ← 1;
(17)     maxFreq ← maxFreq + 1;
```

Figure 10.4: Dynamic ETDC sender pseudo-code.

```
Receiver main algorithm ( )
(1)  new-Symbol ← 1;
(2)  top[0] ← new-Symbol;
(3)  maxFreq ← 1;
(4)  while more codewords remain
(5)       i ← decode(c_i);
(6)       if i =new-Symbol then
(7)            receive s_i in plain form;
(8)            output s_i;
(9)       else
(10)           output word[i];
(11)      update();
```

```
Receiver update ( )
(1)  if i =new-Symbol then // new word
(2)       word[i] ← s_i;
(3)       freq[i] ← 0;
(4)       new-Symbol ← new-Symbol + 1;
(5)  f ← freq[i];
(6)  freq[i] ← freq[i] + 1;
(7)  j ← top[f];
(8)  freq[i] ↔ freq[j];
(9)  word[i] ↔ word[j];
(10) top[f] ← j + 1;
(11) if maxFreq = f + 1 then
(12)      top[f + 1] ← 1;
(13)      maxFreq ← maxFreq + 1;
```

Figure 10.5: Dynamic ETDC receiver pseudo-code.

11

Dynamic (s, c)-Dense Code

This chapter presents the last contribution of this thesis: a new adaptive compression technique called *Dynamic (s,c)-Dense Code* (D-SCDC). D-SCDC is the dynamic version of the (s, c)-Dense Code presented in Chapter 6, and at the same time, it is a generalization of the Dynamic End-Tagged Dense Code, in which not only the vocabulary is maintained sorted dynamically, but also the s and c parameters can vary along the compression/decompression processes in order to achieve a better compression.

The chapter is structured as follows: First the motivation of this new technique is introduced. Section 11.2 describes Dynamic (s, c)-Dense Code. It shows the similarities between the data structures of Dynamic End-Tagged Dense Code and those of Dynamic (s, c)-Dense Code, as well as the differences that arise due to the need to adapt the parameters s and c. Then, two approaches to maintain s and c parameters optimal are presented: the simplest one is shown in Section 11.3 and the most efficient approach is shown in Section 11.4. Section 11.5 shows empirical results where the three new dynamic techniques developed are compared with their corresponding *two-pass* versions, as well as with other well-known compressors such as *gzip*, *bzip2* and an adaptive arithmetic compressor [MNW98]. Lastly, some conclusions are presented in Section 11.6.

11.1 Motivation

In the previous two chapters, two dynamic word-based byte-oriented techniques were defined: the Dynamic word-based byte-oriented Huffman and the Dynamic End-Tagged Dense Code.

The first one is based on Plain Huffman. It achieves better compression than the Dynamic End-Tagged Dense Code (ETDC) (about 1 percentage point in compression ratio). However, it is slower in compression and decompression (about 20%-25% and 45% respectively for large texts).

As shown in Chapter 6, (s, c)-Dense Code [BFNE03] is a compression technique that generalizes End-Tagged Dense Code and obtains compression ratios very close to those of the optimal Plain Huffman Code while using faster and easier-to-program algorithms.

In this chapter, we build an adaptive version of (s, c)-Dense Code. It consists of an extension to Dynamic ETDC where the number of byte values that signal the end of a codeword can be adapted to optimize compression, instead of being fixed at 128 as in Dynamic ETDC.

11.2 Dynamic (s, c)-Dense Codes

Based on Dynamic ETDC, Dynamic (s, c)-DC also uses the scheme presented in Figure 8.1. The *CodeBook* is essentially the vocabulary sorted by frequency, and the on-the-fly *encode* and *decode* procedures, explained in Section 6.4, are used.

The main difference with respect to Dynamic ETDC is that, at each step of the compression/decompression processes, it is mandatory not only to maintain the vocabulary sorted, but also to check whether the current value of s (and consequently the c value) remains optimal or if it should change.

The *update()* algorithm to maintain the sorted list of words is the same used in the case of Dynamic End-Tagged Dense Code (see Figures 10.4 and 10.5). However, the test for a possible change of the s value is new and it has to be performed after calling the update process.

Both encoder and decoder start with $s = 256$. This s value is optimal for the first 255 words of the vocabulary, because it permits encoding all of them with just one byte. When the 256^{th} word arrives, s has to be decreased by 1 since a two-

byte codeword is needed. From this point on, the optimal s and c values will be heuristically estimated.

Two different approaches can be used to keep the optimal values of s and c. The first one, more intuitive, is presented in Section 11.3. It is called *counting bytes approach*. The second alternative, called *ranges approach* is shown in Section 11.4. It gives a more efficient way to maintain optimal the s and c parameters.

11.3 Maintaining optimal the s and c values: *Counting Bytes* approach

The simplest approach to estimate the optimal s and c values as the compression/decompression progresses is based on comparing the size of the compressed text depending on the s and c values used to encode it.

Notice that this approach does not pretend to obtain the absolute optimal s value, but only to follow an efficient heuristical and easy-to-implement approach that still produces a good compression. The general idea is to compare the number of bytes that would be added to the compressed text if the current word w_i would have been encoded using $s-1$, s, and $s+1$. Of course, the value of s, such that it minimizes the size of the compressed text, must be obtained. If the number of bytes needed to encode the word w_i becomes smaller by using either $s-1$ or $s+1$ instead of s, then we update the value of s. Therefore, in each step of the compression/decompression process, the value of s changes at most by one.

Three variables are needed: *prev*, s_0 and *next*. The *prev* variable stores the size of the compressed text assuming that the value $s-1$ was used in the encoding/decoding process. In the same way, s_0 and *next* accumulate the size of the compressed text, assuming that it was encoded using the current s and the $s+1$ values respectively. At the beginning, the three variables are initialized to zero. Each time a word w_i is processed, *prev*, s_0 and *next* are increased.

Let *countBytes(s_value, i)* be the function that computes the number of bytes needed to encode the i^{th} word of the vocabulary with $s = s\_value$ and $c = 256 - s\_value$. Then, intuitively, the three variables are increased as follows:

- $prev \leftarrow prev + countBytes(s-1, i)$
- $s_0 \leftarrow s_0 + countBytes(s, i)$
- $next \leftarrow next + countBytes(s+1, i)$

A change of the s value takes place if $prev < s_0$ or if $next < s_0$. If $prev < s_0$ then $s-1$ becomes the new value of s ($s \leftarrow s-1$). On the other hand, if $next < s_0$ then s is increased ($s \leftarrow s+1$). Notice that it is impossible that both $prev$ and $next$ become smaller than s_0 at the same time, as it is proved in the next two lemmas.

Lemma 11.1 $(s-1)(c+1)^n > sc^n \Rightarrow sc^n > (s+1)(c-1)^n \quad \forall n \geq 1$

Proof We prove it by induction in n. For $n = 1$ we have:

$$(s-1)(c+1) > sc \Rightarrow sc > (s+1)(c-1)$$

from this we get that $s > c+1 \Rightarrow s > c-1$, which clearly holds. Now we assume that:

$(s-1)(c+1)^n > sc^n \Rightarrow sc^n > (s+1)(c-1)^n$

and we prove it for $n+1$

$$(s-1)(c+1)^n(c+1) > sc^n c \Rightarrow sc^n c > (s+1)(c-1)^n(c-1)$$

that is

$c(s-1)(c+1)^n + (s-1)(c+1)^n > sc^n c \Rightarrow sc^n c > c(s+1)(c-1)^n - (s+1)(c-1)^n$

which clearly holds by applying the Induction Hypothesis. □

Lemma 11.2 $W_n^{s-1} > W_n^s \Rightarrow W_n^s > W_n^{s+1} \quad \forall n \geq 2$

Proof We prove it by induction in n. For $n = 2$ we have:

$$(s-1) + (s-1)(c+1) > s + sc \Rightarrow s + sc > (s+1) + (s+1)(c-1)$$

from this we get that $s > c+2 \Rightarrow s > c$, which clearly holds. Now we assume that:

$$W_n^{s-1} > W_n^s \Rightarrow W_n^s > W_n^{s+1}$$

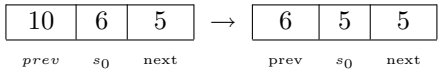
and we prove it for $n+1$

$$W_n^{s-1} + (s-1)(c+1)^n > W_n^s + sc^n \Rightarrow W_n^s + sc^n > W_n^{s+1} + (s+1)(c-1)^n$$

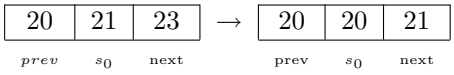
which holds by the Induction Hypothesis and Lemma 11.1. □

Therefore, it is impossible that $prev < s_0 > next$, and hence we can easily decide in which direction s must be modified. Each time the parameter s changes, the values $prev$, s_0, and $next$ are initialized again, and then the process continues. This initialization depends on the change of s that takes place. In order to keep the history of the process we do not initialize the three values to zero, but we use the previous values. Of course, one of the three values (either $prev$ or $next$, depending on the direction of the change of s) is unknown and we set it to the same value of s_0. Summarizing:

- If s is increased then $prev \leftarrow s_0$ and $s_0 \leftarrow next$ ($next$ does not change).



- If s is decreased then $next \leftarrow s_0$ and $s_0 \leftarrow prev$ ($prev$ does not change).



There are other alternatives for this basic algorithm. For example, it would be possible to use an ε value as a threshold for the change of s. That is, the value of s would change only if $prev + \varepsilon < s_0$ or $next + \varepsilon < s_0$. In this way, less changes would take place and the algorithm would be a little faster, but the compression ratio would not be so good.

Another possible choice would be to initialize the three variables $prev$, s_0, and $next$ to *zero* each time the value of s changes. This choice would make the algorithm free from the previous history. This approach can be interesting in natural language documents where the vocabulary, and consequently its frequency distribution, changes frequently along the text. On the other hand, assuming that the document is rather homogeneous, keeping the previous knowledge seems interesting. We do that by initializing s_0 with the value of either $prev$ or $next$ and then keeping the former value of s_0 in either $next$ or $prev$ respectively, depending on whether we are increasing or decreasing the value of s.

11.3.1 Pseudo-code for the *Counting Bytes* approach

Two main procedures are used in this *counting bytes* approach to check whether the s and c values should change after having processed a new source symbol. The first one is the already described *countBytes()* algorithm.

```
countBytes Algorithm (s_i, i_pos)
    //Calculates the length of the codeword assigned to a word ranked i_pos
    //Using s_i as value of s
(1) k ← 1;
(2) last ← s_i;
(3) pot ← s_i;
(4) c_i ← 256 − s_i;
(5) while last ≤ i_pos
(6)       pot ← pot × c_i;
(7)       last ← last + pot;
(8)       k ← k + 1;
(9) return k;
```

```
TakeIntoAccount Algorithm (s, c, i)
    //It changes, if needed, the s and c values
(1) prev ← countBytes (s − 1, i);
(2) s_0 ← countBytes (s, i);
(3) if prev < s_0 then
(4)       s ← s − 1; //s is decreased
(5)       c ← c + 1;
(6)       next ← s_0;
(7)       s_0 ← prev;
(8) else
(9)       next ← countBytes (s + 1, i);
(10)      if next < s_0 then
(11)            s ← s + 1; //s is increased
(12)            c ← c − 1;
(13)            prev ← s_0;
(14)            s_0 ← next;
```

Figure 11.1: Algorithm to change parameters s and c.

The second procedure, called *TakeIntoAccount()*, is invoked after each call to the *update()* procedure. It first uses *countBytes()* to increase *prev*, s_0 and *next*. Then, it checks either if $prev < s_0$ or if $next < s_0$, and changes the s value used in compression or decompression if necessary.

The pseudo-code for both algorithms is shown in Figure 11.1. Notice that the *countBytes()* algorithm is called at least twice in each execution of the *TakeIntoAccount()* algorithm. The cost of *countBytes()* depends on the *maximum codeword length*. It is $O(\log_c(n))$ if $c > 1$, and $O(\frac{n}{255})$ if $c = 1$. However, in practice the value $c = 1$ is only used when the size of the vocabulary is between 256 and 510 (the c value increases rapidly while the first thousands of words are processed). Since the maximum codeword length for $s = 255$, $c = 1$ and $n < 510$ is 2, it can be

considered that the cost of *countBytes()* is $O(\log_c(n))$.

Since *TakeIntoAccount()* is called N times (being N the number of source words), the cost of the whole process is $O(N\ \log_c(n))$. Actually, this is proportional to the number of bytes of the compressed stream.

As a result, maintaining optimal the s and c values can be done quite efficiently. It can also be seen that the *countBytes()* algorithm takes an important part of the time spent by *TakeIntoAccount()*. In the following section, a more efficient way to maintain optimal the s and c values is presented. It consists of a more sophisticated alternative that avoids using *countBytes()*, by using some knowledge about how the values W_k^s behave.

11.4 Maintaining optimal the s and c values: *Ranges* approach

This approach is based on an interesting property that arises from the definition of W_k^s in Section 6.2. The key idea is to use the ranges defined by the different W_k^s, for $128 \leq s \leq 256$ and $1 \leq k \leq 4$, in the rank of the words of the vocabulary. These ranges indicate the number of bytes k that are needed to encode a word w_i, (i is the rank in the vocabulary for w_i) using $s-1$, s, and $s+1$ stoppers. That is, these ranges allow us to know, for any word w_i and using a specific s value, whether encoding that word with $s-1$ or $s+1$ stoppers would produce codewords with equal, more, or less bytes. The main difference with the previous approach is that now it is not necessary to use the time-consuming *CountBytes(s, i)* procedure to know the number of bytes of the codeword associated to w_i if $s-1$ or $s+1$ were used instead of s.

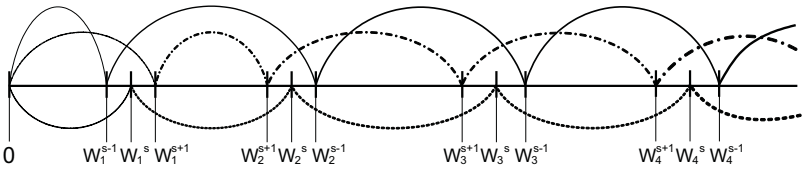


Figure 11.2: Ranges defined by W_k^{s-1}, W_k^s and W_k^{s+1}.

We do not consider values of s smaller than 128 because, as shown in Section 6.3.1, those values never improve compression ratios in natural language applications were the number of words in the vocabulary is never large enough to

take advantage of the large amount of words that can be encoded with 4, 5, 6, or more bytes. More specifically, if $n < 2,113,664$ then values of s such that $s < 128$, are never convenient, and using codewords with up to 4 bytes ($k <= 4$) is always enough.

Figure 11.2 shows those ranges. The points W_k^{s-1}, W_k^s and W_k^{s+1}, for $1 \leq k \leq 4$ and $s > c$, are represented. Notice that words in $[0,\ s-1)$, $[0,\ s)$ or $[0,\ s+1)$ can be encoded with a unique byte, if the values $s-1$, s, or $s+1$ respectively are used. In the same way, words in ranges $[s-1,W_2^{s-1})$, $[s,W_2^s)$, or $[s+1,W_2^{s+1})$ are encoded with *two-byte* codewords when the values $s-1$, s, or $s+1$ respectively are used, and so on.

Notice that if, for example $W_2^s \leq i < W_2^{s-1}$, then the word w_i would be encoded with a 3-byte codeword using s but with a 2-byte using $s-1$. In the same way, if $W_2^{s-1} \leq i < W_3^{s+1}$, then a 3-byte codeword will be assigned to the word w_i when any of the three s, $s-1$, or $s+1$ values is used.

In Figure 11.2, we carefully place the different values W_k^s, W_k^{s-1}, and W_k^{s+1} to keep the real relationships among them. Notice that the order changes from $W_1^{s-1} \leq W_1^s \leq W_1^{s+1}$, to the opposite $W_k^{s+1} \leq W_k^s \leq W_k^{s-1} \quad \forall k \geq 2$. Next we prove that those relationships shown in the figure actually hold when $s > c$.

Lemma 11.3 *If $s > c$, it holds that:*

a) $W_1^{s+1} > W_1^s > W_1^{s-1}$

b) $W_k^{s+1} \leq W_k^s \leq W_k^{s-1}$, $\forall k \geq 2$

Proof The case *a)*, $(k = 1)$ is trivial from the definition of W_k^s, since $W_1^{s-1} = s-1$, $W_1^s = s$ and $W_1^{s+1} = s+1$.

In order to prove part *b)* we only need to prove that $W_k^s \leq W_k^{s-1}$, $\forall s > c$. We proceed by induction in k. For $k = 2$ we prove that:

$$W_k^s \ < \ W_k^{s-1}$$

that is,

$$s \ + \ sc \ < \ (s-1) \ + \ (s-1)(c+1)$$

this is reduced to $s \geq c+2$, which is always true when $s > c$ because $s + c = 256$. That is, the minimum value of s such that $s > c$ is $s = 129$. In such case, $c = 127$ accomplishes that $s \geq c+2$.[1]

[1] This remains true for any $s + c = 2^b$, since we always want to use a fixed number of b bits per output symbol. The property only depends on 2^b being even.

We assume by Induction Hypothesis that $W_k^s \leq W_k^{s-1}$ and we prove the property for the case $k+1$

$$W_k^s + sc^k \leq W_k^{s-1} + (s-1)(c+1)^k$$

which holds from the Induction Hypothesis and Lemma 11.4. □

Lemma 11.4 $sc^k \leq (s-1)(c+1)^k, \quad \forall s > c.$

Proof We proceeded by induction in k. For $k = 1$ we have:

$$sc \leq (s-1)(c+1)$$

that is,

$$c + 1 \leq s$$

which holds since $s > c$ and both are integers. Assuming that the lemma holds for k, (that is, assuming $sc^k < (s-1)(c+1)^k$) we prove it for the case $k+1$:

$$sc^k c \ \leq \ (s-1)(c+1)^k(c+1)$$

that is,

$$sc^k c \ \leq \ c((s-1)(c+1)^k) + (s-1)(c+1)^k$$

This expression clearly holds from the Induction Hypothesis and the fact that $0 \leq (s-1)(c+1)^k$. □

To end this introductory section we illustrate in Figure 11.3 how the s value evolves in practice, as the compression process progresses.

In Figure 11.3, it can be seen that the dynamic encoder adapts the s value rapidly in order to reduce the codeword length. Therefore, the s value falls from 256 to a value close to 129 when the first $16,512$ words are processed. When $n > 16,512$, three-byte codewords are needed, therefore the s value is increased. Fluctuations of s beyond that point depend on the order in which words are input (and also on their frequencies).

11.4.1 General description of the Ranges approach

Lemma 11.3 permits maintaining the optimal s and c values in a more efficient fashion than that presented in Section 11.3.

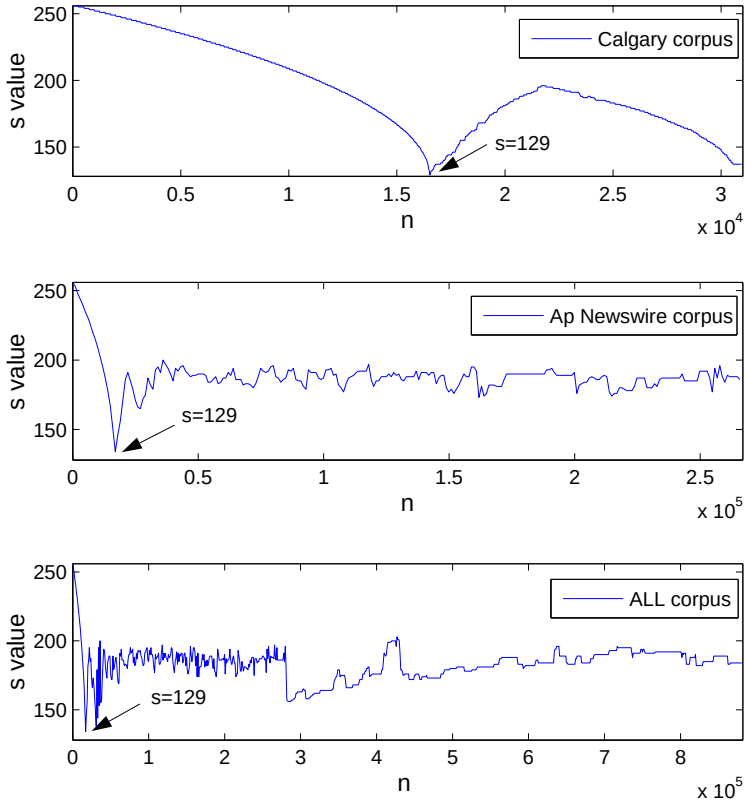


Figure 11.3: Evolution of s as the vocabulary grows.

Assume that the word ranked i is being encoded (or decoded) using the second on-the-fly algorithm showed in Section 6.4. Notice that this algorithm returns not only the actual codeword but also its length. Let k be the length of the codeword, then the specific interval in the rank of words that needs to be considered is $[W_k^s, W_{k+1}^s)$.

Three subintervals need to be considered inside of this interval. Table 11.1 represents (columns 1 and 4) the limits of each interval $[W_k^s, W_{k+1}^s)$ for $1 \leq k \leq 4$. Inside of each interval, the two internal limits that mark the three internal subintervals (columns 2 and 3 in the table) are shown. Notice that, in different format, the same subintervals are represented in Figure 11.2.

The left hand side subinterval (that does not exist when $k = 1$) always represents

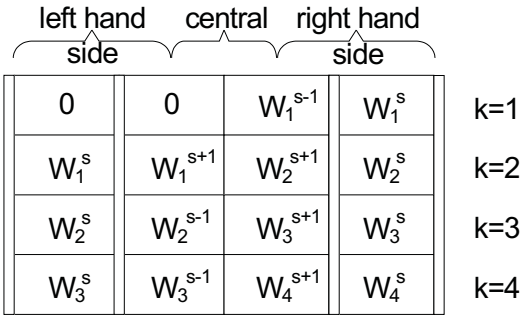


Table 11.1: Subintervals inside the interval $[W_k^s, W_{k+1}^s)$, for $1 \leq k \leq 4$.

word ranks that are encoded with a one-byte-shorter codeword if either $s+1$ (when $k = 2$) or $s - 1$ (when $k > 2$) are used instead of s. The central subinterval, the one defined by the two central values of each row in the table, represents word ranks that are encoded with codewords of the same length, no matter if s, $s-1$, or $s+1$ is used. Finally, the right hand side subinterval represents word ranks that are encoded with a one-byte-longer codeword if $s - 1$ (when $k = 1$) or $s + 1$ (when $k > 1$) are used instead of s.

The algorithm uses a pre-computed three-dimensional matrix keeping the values for the subinterval limits for $128 \leq s \leq 256$ and $1 \leq k \leq 4$. In this way, all the necessary ranges are predefined. The algorithm easily and efficiently finds out whether $s-1$ or $s+1$ would compute one-byte-longer or one-byte-shorter codeword than s, for the current word w_i, without using the time-consuming *countBytes(s,i)* algorithm.

In the *Counting Bytes* approach we accumulated in *prev*, s_0 and *next* the size of the compressed text by adding for each word the number of bytes of the codewords that would be computed if $s-1$, s, and $s+1$ were used. In the *Ranges* approach, for each word w_i, we only add $+1$ as a *penalty* if i is in the right hand side subinterval, or -1 as a *bonus* if i is in the left hand side subinterval. This addition of $+1$ or -1 is done in *prev* or in *next* depending on the value of k.

Clearly if i is in a central subinterval no action has to be performed. If i is in a right hand side subinterval, a penalty is added to *prev* (when $k = 1$) or to *next* (when $k > 1$), therefore s is still optimal and it is not necessary to check if there is a better value for s. But if i is in a left hand side subinterval then a bonus -1 is added to *next* (when $k = 2$) or to *prev* (when $k > 2$) and in this case, it is necessary to check if s is still optimal by comparing s_0 with *prev* or *next* (depending on which

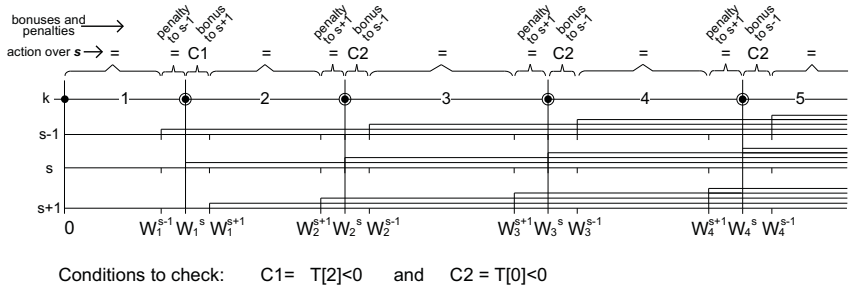


Figure 11.4: Intervals and subintervals, penalties and bonus.

one of them has received the bonus).

Figure 11.4 represents in the first (top-most) row the places in the rank of words where penalties and bonuses are added either to *prev* or to *next*. The second row shows the action that is performed on the value of s. The equal sign represents that s is still optimal for sure and, therefore, it is not necessary to check it. In the same row, the places where s must be checked and the test that need to be performed are shown. The third row represents the different intervals that, as it was already explained, correspond to the k values. The different number of bytes needed to encode a word w_i using s, $s-1$, and $s+1$ are also shown in Figure 11.4. Notice that the number of horizontal lines correspond to the number of bytes needed to encode each codeword with the corresponding value of s. Next section presents all the implementation details.

11.4.2 Implementation

Three elements have to be taken into account: *i)* The definition of the ranges that correspond to the current s value, *ii)* the penalties or bonuses that have to be applied in some cases to the use of $s-1$ or $s+1$, and *iii)* the action that takes place on the s value (increasing or decreasing it).

Data Structures

One vector T, and two matrices I and V are needed. They are defined as follows:

- Vector T. A 3-element vector that corresponds to the *prev*, s_0 and *next* variables (in vector form for implementation reasons). Each position $T[i]$, $0 \leq$

$i \leq 2$, stores a signed number that accumulates bonuses and penalties for the values $s-1$, s, and $s+1$ respectively. Bonuses decrease by 1 either the value $T[0]$ or $T[2]$. Penalties increase those values by 1. If $T[0]$ becomes negative, it means that $s-1$ is preferable to the current value s, since it received more bonuses than penalties. On the opposite, if $T[2] < 0$ then the action that follows is to change s to $s+1$. As an analogy with the first approach presented in Section 11.3, $T[0]$ and $T[2]$ are *prev* and *next* values respectively. However, in this case, $T[0]$ and $T[2]$ do not accumulate the estimated *size of the compressed text*, but the extra number of bytes needed when either $s-1$ or $s+1$ are used instead of s. Notice that $T[1] = 0$ always and therefore it is not really necessary (we keep it here for clarity).

$$T = \begin{array}{|c|c|c|} \hline \pm x & 0 & \pm y \\ \hline \end{array}$$

- Matrix I is a pre-computed three dimensional matrix with size $128 \times 4 \times 2$. This matrix represents, for each value of s ($128 \leq s \leq 256$) and for each value of k ($1 \leq k \leq 4$) the two limits of the subintervals inside the interval defined by s and k. Remember that this interval is $[W_k^s, W_{k+1}^s)$. The subinterval limits need to be checked each time a word w_i is processed to know whether the codeword length would be a byte longer or shorter if $s-1$ or $s+1$ were used instead of s to encode w_i. When a word w_i is processed, we first encode it using the second on-the-fly encoding algorithm shown in Section 6.4. This algorithm returns not only the codeword but also its length. Then it is necessary to check whether i is in the left, central, or right subinterval to determine the action that should be taken over $T[0]$ or $T[2]$ (bonus or penalty). That is, if $i < I[s,k,0]$ then it is in the left hand side subinterval and a bonus need to by added. If $i \geq I[s,k,1]$ then a penalty will be added. Otherwise i is in the central subinterval, and neither a penalty nor a bonus are necessary, since the codeword length would be the same with $s-1$, s, and $s+1$.

  Next table shows the section of the matrix I for an specific s value. Certainly, matrix I has 128 sections as the one showed, one for each possible value of s.

$$I_s = \begin{array}{|c|c|} \hline \text{bonuses} < & > \text{penalties} \\ \hline 0 & W_1^{s-1} \\ \hline W_1^{s+1} & W_2^{s+1} \\ \hline W_2^{s-1} & W_3^{s+1} \\ \hline W_3^{s-1} & W_4^{s+1} \\ \hline \end{array} \begin{array}{l} \longleftarrow k=1 \\ \longleftarrow k=2 \\ \longleftarrow k=3 \\ \longleftarrow k=4 \end{array}$$
$$\begin{array}{cc} 0 & 1 \end{array}$$

- Matrix V is a 4×2 predefined table. Each cell can contain either 0 or 2, and this value is used to index the vector T. That is, we use this table to indicate

whether $T[0]$ or $T[2]$ is the one to receive the bonus or the penalty. The first row represents penalties, so it is used when a penalty has to be added to $T[0]$ or to $T[2]$. In the same way, the second row is used for the bonuses. After using matrix I to know if a penalty or a bonus has to be added, we use this matrix to know whether it must be added either to $T[0]$ or to $T[2]$. The matrix V has the following values:

V =

$k=1$ ↓	$k=2$ ↓	$k=3$ ↓	$k=4$ ↓	
–	2	0	0	*bonuses*
0	2	2	2	*penalties*

For example if $k = 2$ and if using the I matrix, the algorithm determines that a bonus has to be added, then the algorithm performs $T[V[0,2]] \longleftarrow T[V[0,2]] - 1$, and after that, a comparison between $T[V[0,2]]$ and $T[1]$ is needed to check if s is still optimal. This comparison is performed by simply checking if $T[V[0,2]] < 0$.

If, for example, $k = 4$ and a penalty has to be added, the algorithm performs $T[V[1,4]] \longleftarrow T[V[1,4]]+1$. Now it is not necessary to check if $T[V[1,4]] < 0$. Since the value $T[V[1,4]]$ was increased, it cannot have become a negative value. Therefore, we are sure that s is still optimal.

Obviously, if the algorithm determines, using the matrix I, that it is not necessary to add neither a bonus nor a penalty, no further action is performed, and the table V is not used in that step.

Pseudo-code for the *Ranges* approach

The pseudo-code for the algorithm that handles the update of the s and c values following this approach is presented in Figure 11.5. Such algorithm is called *TakeIntoAccountRanges()*.

Assuming that the i^{th} word of the vocabulary is being encoded/decoded, the on-the-fly encoding algorithm encodes the word w_i, returning the codeword and its length k. The decoding algorithm returns the position in the vocabulary i, and the number of bytes (k) of the codeword decoded. Then, the algorithm *TakeIntoAccountRanges()* knows the values k, i, and s. Using matrix I, it looks for the subinterval (range) where i appears, in order to know if a bonus or a penalty has to be added. In such case, using the matrix V the algorithm finds if the bonus or the penalty must be added to either $T[0]$ or $T[2]$. Finally, if a bonus was added to either $T[0]$ or $T[2]$, one more check has to be done: if $T[0]$ was the decreased

term, then it is checked if $T[0] < 0$; in this case, s is set to $s-1$. On the other hand, if $T[2]$ was the decreased term, then it is checked if it became a negative value. In such case, the s value is increased. Each time s changes, the value of $T[0]$ and $T[2]$ is initialized following the same strategy described in the *Count Bytes* approach.

```
TakeIntoAccountRanges Algorithm (s, k, i)
    //It changes, if needed, the s and c values used in both compression
    //and decompression processes. It checks the i range and the addition
    //of either bonuses or penalties.
(1) if i ≥ I[s, k, 1] then
(2)     T[V[1, k]] ← T[V[1, k]] + 1;
(3) else
(4)     if i < I[s, k, 0] then
(5)         T[V[0, k]] ← T[V[0, k]] − 1;
            //Checking if s is still optimal
(6)         if T[V[0, k]] < 0 then
(7)             s ← s + V[0, k] − 1  //change of s
(8)             c ← 256 − s;
(9)             T[V[0, k]] ← 0;
(10)            T[2 − V[0, k]] ← 1;
```

Figure 11.5: Algorithm to change parameters s and c.

Observe, in lines 9 and 10, how $T[0]$ and $T[2]$ are initialized after any change of s. Remember that $T[1]$ never needs to be changed because it is always 0. The idea is to keep only one unit of difference between the new $T[1]$ and the former one, but instead of initializing $T[1]$ to -1 and $T[2-V[0,k]]$[2] to 0, we prefer to initialize $T[2-V[0,k]]$ to 1 and to maintain $T[1] = 0$. In the same way, following the same strategy described in the *Count Bytes* approach, we initialize $T[V[0,k]]$ with the same value of $T[1]$, that is, with 0. Remember that in the *Count Bytes* approach, we initialize *prev* or *next* (depending on the direction of the change) with the same value of s_0.

11.5 Empirical results

We first compressed the texts in our experimental framework to compare the three dynamic techniques among them in compression ratio and in compression and decompression speed. Those results are shown in Section 11.5.1. Next, the compression ratio and time performance of the one- and two-pass versions of End-Tagged Dense Code (ETDC), (s,c)-Dense Code (SCDC), and Plain Huffman (PH)

[2]Notice that $T[2-V[0,k]]$ represents $T[0]$ or $T[2]$ depending on the value of $V[0,k]$.

are shown in Section 11.5.2. Finally, we also compressed all the corpora with other three well-known compression techniques: *gzip*, *bzip2*, and a word-based arithmetic compressor [CMN+99]. Results showing compression and decompression speed, and also compression ratio for all these techniques are presented in Section 11.5.3.

11.5.1 Dynamic approaches: compression ratio and time performance

Table 11.2 compares the time performance and the compression ratio obtained by our three dynamic compressors. For each technique, the first sub-table shows the compression ratio, the compression time, and the decompression time needed to decompress each corpus.

The second sub-table included in Table 11.2 compares both Dynamic ETDC and Dynamic SCDC with respect to Dynamic Plain Huffman. Columns two and three measure differences in compression ratio. The fourth and the fifth columns show the reduction of compression time (in percentage) of Dynamic ETDC and Dynamic SCDC, with respect to Dynamic PH. Finally, the last two columns in the second sub-table show the reduction in decompression time obtained by Dynamic ETDC and Dynamic SCDC.

As it can be seen, Dynamic ETDC loses less than 1 point in compression ratio (about 3% increase of size) with respect to Dynamic Plain Huffman, in the larger texts. In exchange, it is 20%-25% faster in compression and reduces its decompression time by about 45%. Moreover, Dynamic ETDC is considerably simpler to implement.

In the case of Dynamic SCDC the loss of compression ratio with respect to Dynamic Plain Huffman is around 0.25 percentage points. The compression speed is improved by about 17% and decompression time is reduced over 39%.

It is noticeable that, differences in compression and decompression speed between Dynamic PH and our dynamic "dense" codes increase more and more as the size of the text collection grows. Those differences arise because the update algorithm is much simpler (and faster) in the "dense" techniques.

The results also show that Dynamic ETDC is a bit faster than the Dynamic SCDC. This is due to two main issues: *i)* the encoding process uses faster bitwise operations (in the case of Dynamic ETDC), and *ii)* Dynamic SCDC has to use the TakeIntoAccount algorithm to maintain optimal the parameters s and c, whereas those values are fixed to 128 in Dynamic ETDC.

CORPUS	Compr. ratio			Compr. time (sec)			Decompr. time (sec)		
	Dyn PH	Dyn SCDC	Dyn ETDC	Dyn PH	Dyn SCDC	Dyn ETDC	Dyn PH	Dyn SCDC	Dyn ETDC
CALGARY	46.546	46.809	47.730	0.342	0.292	0.268	0.216	0.135	0.122
FT91	34.739	34.962	35.638	2.322	1.940	1.788	1.554	0.942	0.847
CR	31.102	31.332	31.985	7.506	6.532	6.050	5.376	3.272	3.033
FT92	32.024	32.237	32.838	29.214	24.440	22.905	21.726	12.287	11.603
ZIFF	32.895	33.078	33.787	30.218	26.057	24.042	21.592	12.815	11.888
FT93	32.005	32.202	32.887	32.506	27.682	25.505	23.398	13.608	12.675
FT94	31.959	32.154	32.845	33.310	28.817	26.385	23.802	13.837	13.013
AP	32.294	32.557	33.106	43.228	36.113	33.700	32.184	17.623	16.923
ALL_FT	31.710	31.849	32.537	103.354	85.197	79.307	75.934	42.055	39.970
ALL	32.849	33.029	33.664	209.476	167.403	157.277	161.448	87.488	81.893

CORPUS	Diff. Ratio		Decr. Compr. time (%)		Decr. Decomp. Time (%)	
	Dyn SCDC	Dyn ETDC	Dyn SCDC	Dyn ETDC	Dyn SCDC	Dyn ETDC
CALGARY	0.263	1.184	14.717	21.540	37.500	43.673
FT91	0.223	0.899	16.451	22.983	39.404	45.517
CR	0.229	0.883	12.981	19.398	39.143	43.576
FT92	0.214	0.814	16.341	21.596	43.447	46.592
ZIFF	0.183	0.891	13.771	20.439	40.649	44.941
FT93	0.197	0.882	14.841	21.538	41.840	45.829
FT94	0.195	0.887	13.489	20.790	41.868	45.327
AP	0.263	0.812	16.458	22.041	45.242	47.417
ALL_FT	0.139	0.827	17.568	23.267	44.616	47.362
ALL	0.180	0.815	20.085	24.919	45.810	49.276

Table 11.2: Comparison among our three dynamic techniques.

To sum up, the compression speed obtained by Dynamic ETDC, Dynamic SCDC, and Dynamic PH is, respectively, about 5, 6.5 and 7 Mbytes per second. In decompression, the differences obtained are more spectacular. In general, the decompression speed obtained by Dynamic ETDC, Dynamic SCDC, and Dynamic PH is, respectively, over 15, 14, and 8 Mbytes per second.

11.5.2 Semi-static Vs dynamic approach

Comparison in compression ratio

Table 11.3 compares the compression ratios of the *two-pass* versus the *one-pass* versions of ETDC, SCDC and PH. Columns labelled $\mathbf{diff}_{PH}$, $\mathbf{diff}_{ETDC}$ and $\mathbf{diff}_{SCDC}$ measure the increase, in percentage points, in the compression ratio of the dynamic codes compared with their semi-static version. The last two columns compare the differences in the increments of compression ratio.

As it can be seen, a very small loss of compression occurs, in the three techniques, when they are implemented as dynamic codes. To understand this increase in size of dynamic versus semi-static codes, two issues have to be considered: (*i*) each new word s_i parsed during dynamic compression is represented in the compressed text (or sent to the receiver) as a pair $\langle C_{new\text{-}Symbol}, s_i \rangle$, while in two-pass compression only the word s_i needs to be stored/transmitted in the vocabulary; (*ii*) the codewords used to transmit words can be shorter or larger than the optimal

CORPUS	TEXT SIZE	Plain Huffman			End-Tagged Dense Code		
		2-pass	dynamic	Increase	2-pass	dynamic	Increase
	bytes	ratio	ratio	diff_{PH}	ratio	ratio	diff_{ETDC}
CALGARY	2,131,045	46.238	46.546	0.308	47.397	47.730	0.332
FT91	14,749,355	34.628	34.739	0.111	35.521	35.638	0.116
CR	51,085,545	31.057	31.102	0.046	31.941	31.985	0.045
FT92	175,449,235	32.000	32.024	0.024	32.815	32.838	0.023
ZIFF	185,220,215	32.876	32.895	0.019	33.770	33.787	0.017
FT93	197,586,294	31.983	32.005	0.022	32.866	32.887	0.021
FT94	203,783,923	31.937	31.959	0.022	32.825	32.845	0.020
AP	250,714,271	32.272	32.294	0.021	33.087	33.106	0.018
ALL_FT	591,568,807	31.696	31.710	0.014	32.527	32.537	0.011
ALL	1,080,719,883	32.830	32.849	0.019	33.656	33.664	0.008

CORPUS	(s, c)-Dense Code			diff_{ETDC}	diff_{SCDC}
	2-pass	dynamic	Increase	$-\ \text{diff}_{PH}$	$-\ \text{diff}_{PH}$
	ratio	ratio	diff_{SCDC}		
CALGARY	46.611	46.809	0.198	0.024	-0.110
FT91	34.875	34.962	0.086	0.005	-0.025
CR	31.291	31.332	0.041	-0.001	-0.005
FT92	32.218	32.237	0.020	-0.001	-0.004
ZIFF	33.062	33.078	0.016	-0.002	-0.003
FT93	32.178	32.202	0.024	-0.001	0.002
FT94	32.132	32.154	0.021	-0.002	-0.001
AP	32.542	32.557	0.015	-0.003	-0.006
ALL_FT	31.839	31.849	0.010	-0.003	-0.004
ALL	33.018	33.029	0.011	-0.011	-0.008

Table 11.3: Compression ratio of dynamic versus semi-static techniques.

codeword that is assigned in the semi-static version. Even some gain in compression can be obtained by occasional shorter codewords assigned to low frequency words (especially when they appear at the beginning of the text), however, in general some loss in compression appears because the optimal codeword is not always used over the whole text.

The compression ratios obtained by the three techniques are around 31%-33% for large texts. For the smallest one (Calgary collection), compression is poorer because the size of the vocabulary is proportionally large with respect to the compressed text size (as expected from Heaps' law, that is described in Section 2.4.1).

The dynamic versions of the compressors lose very little compression (not more than 0.05 percentage points in general) compared to their semi-static versions in the three techniques. This shows that the price paid by dynamism in terms of compression ratio is practically negligible. Note also that, in most cases, dynamic ETDC loses even less compression than dynamic Plain Huffman. In the same way, adding dynamism to (s,c)-Dense Code only produces more loss of compression ratio than dynamic PH in the FT93 corpus.

Comparison in compression and decompression speed

In this section, we compare the compression and decompression time (measured in seconds) obtained by our new dynamic techniques and their corresponding semi-

CORPUS	Semi-static approach (sec)			Dynamic approach (sec)			Loss in speed (%)		
	PH	SCDC	ETDC	PH	SCDC	ETDC	PH	SCDC	ETDC
CALGARY	0.415	0.405	0.393	0.342	0.292	0.268	21.345	38.857	46.584
FT91	2.500	2.493	2.482	2.322	1.940	1.788	7.666	28.522	38.770
CR	7.990	7.956	7.988	7.506	6.532	6.050	6.448	21.810	32.029
FT92	29.243	29.339	29.230	29.214	24.440	22.905	0.098	20.044	27.614
ZIFF	30.354	30.620	30.368	30.218	26.057	24.042	0.451	17.513	26.312
FT93	32.915	33.031	32.783	32.506	27.682	25.505	1.258	19.326	28.535
FT94	33.874	33.717	33.763	33.310	28.817	26.385	1.692	17.004	27.961
AP	42.641	42.676	42.357	43.228	36.113	33.700	-1.357	18.172	25.689
ALL_FT	99.889	100.570	100.469	103.354	85.197	79.307	-3.353	18.045	26.684
ALL	191.396	191.809	191.763	209.476	167.403	157.277	-8.631	14.579	21.927

a) Compression time.

CORPUS	Semi-static approach (sec)			Dynamic approach (sec)			Gain in speed (%)		
	PH	SCDC	ETDC	PH	SCDC	ETDC	PH	SCDC	ETDC
CALGARY	0.088	0.097	0.085	0.216	0.135	0.122	59.105	28.395	30.137
FT91	0.577	0.603	0.570	1.554	0.942	0.847	62.891	35.929	32.677
CR	1.903	1.971	1.926	5.376	3.272	3.033	64.611	39.752	36.520
FT92	7.773	7.592	7.561	21.726	12.287	11.603	64.225	38.209	34.837
ZIFF	8.263	7.988	7.953	21.592	12.815	11.888	61.730	37.669	33.104
FT93	8.406	8.437	8.694	23.398	13.608	12.675	64.075	38.000	31.410
FT94	8.636	8.690	8.463	23.802	13.837	13.013	63.716	37.196	34.971
AP	11.040	11.404	11.233	32.184	17.623	16.923	65.697	35.292	33.622
ALL_FT	24.798	25.118	24.500	75.934	42.055	39.970	67.343	40.275	38.704
ALL	45.699	46.698	46.352	161.448	87.488	81.893	71.695	46.624	43.400

a) Decompression time.

Table 11.4: Time performance in semi-static and dynamic approaches.

static codes.

Table 11.4.a) gives a comparison in compression time. For each corpus, columns two, three, and four show the compression time obtained by the semi-static codes. The next three columns present compression time for the dynamic compressors. The last three columns in that table show the loss of compression speed of the semi-static codes with respect to the dynamic ones. The dynamic techniques obtain, in general, better compression speed than the two-pass ones. The dynamic PH is only faster than Plain Huffman in small and medium-size corpora, whereas dynamic ETDC and dynamic SCDC improve the compression speed of ETDC and SCDC in all text collections. As it already happened in the two previous chapters, the differences between both semi-static and dynamic approaches decrease as the size of the corpus grows. In general, the slower and more complex update algorithm, the smaller the differences between both the semi-static and the dynamic approaches. This can be observed in Figure 11.6.

Table 11.4.b) presents the results regarding to decompression time. The last three columns in that table show the gain in decompression speed of the two-pass techniques with respect to the dynamic ones. It can be seen that, in decompression, the semi-static techniques are clearly faster than the dynamic decompressors. In this case, differences in time between both approaches grow as the size of the corpora increases. Figure 11.6 summarizes those results.

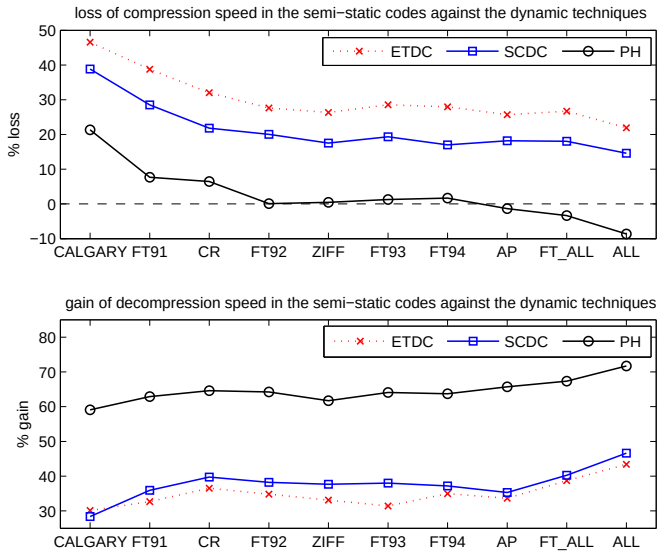


Figure 11.6: Progression of compression and decompression speed.

11.5.3 Comparison against other adaptive compressors

Table 11.5 compares Dynamic PH, Dynamic SCDC and Dynamic ETDC against *gzip* (Ziv-Lempel family), *bzip2* (Burrows-Wheeler [BW94] type technique) and a word-based adaptive arithmetic compressor (arith)[CMN+99]. Experiments were run setting *gzip* and *bzip2* parameters to both "best" (-b) and "fast" (-f) compression.

As expected, "bzip2 -b" achieves the best compression ratio. It is about 5-7 percentage points better than Dynamic PH (and hence a little bit more with respect to Dynamic SCDC and Dynamic ETDC). However, it is much slower than the other techniques tested in both compression and decompression. Using the "fast" *bzip2* option seems to be undesirable, since compression ratio gets worse (becoming closer to Dynamic PH) and compression and decompression speeds remain poor.

With respect to gzip we found that "gzip -f" achieves good compression speed, at the expense of poor compression ratio (about 40%). It is shown that Dynamic ETDC is the fastest compressor. In fact, Dynamic ETDC is able to beat "gzip -f" in compression speed in all corpora. Dynamic SCDC is a bit slower than the Dynamic ETDC, and overcomes also the compression speed obtained by "gzip -f" (over 10%). Moreover, as in the case of ETDC, it is much better in compression

CORPUS	compression ratio							
	D-PH	D-SCDC	D-ETDC	arith	gzip -f	gzip -b	bzip2 -f	bzip2 -b
CALGARY	46.546	46.809	47.730	34.679	43.530	36.840	32.827	28.924
FT91	34.739	34.962	35.638	28.331	42.566	36.330	32.305	27.060
CR	31.102	31.332	31.985	26.301	39.506	33.176	29.507	24.142
FT92	32.024	32.237	32.838	29.817	42.585	36.377	32.369	27.088
ZIFF	32.895	33.078	33.787	26.362	39.656	32.975	29.642	25.106
FT93	32.005	32.202	32.887	27.889	40.230	34.122	30.624	25.322
FT94	31.959	32.154	32.845	27.860	40.236	34.122	30.535	25.267
AP	32.294	32.557	33.106	28.002	43.651	37.225	33.260	27.219
ALL_FT	31.710	31.849	32.537	27.852	40.988	34.845	31.152	25.865
ALL	32.849	33.029	33.664	27.982	41.312	35.001	31.304	25.981

CORPUS	compression time (sec)							
	D-PH	D-SCDC	D-ETDC	arith	gzip -f	gzip -b	bzip2 -f	bzip2 -b
CALGARY	0.342	0.292	0.268	1.030	0.360	1.095	2.180	2.660
FT91	2.322	1.940	1.788	6.347	2.720	7.065	14.380	18.200
CR	7.506	6.532	6.050	21.930	8.875	25.155	48.210	65.170
FT92	29.214	24.440	22.905	80.390	34.465	84.955	166.310	221.460
ZIFF	30.218	26.057	24.042	82.720	33.550	82.470	174.670	233.250
FT93	32.506	27.682	25.505	91.057	36.805	93.135	181.720	237.750
FT94	33.310	28.817	26.385	93.467	37.500	96.115	185.107	255.220
AP	43.228	36.113	33.700	116.983	50.330	124.775	231.785	310.620
ALL_FT	103.354	85.197	79.307	274.310	117.255	293.565	558.530	718.250
ALL	209.476	167.403	157.277	509.710	188.310	532.645	996.530	1,342.430

CORPUS	Decompression time (sec)							
	D-PH	D-SCDC	D-ETDC	arith	gzip -f	gzip -b	bzip2 -f	bzip2 -b
CALGARY	0.216	0.135	0.122	0.973	0.090	0.110	0.775	0.830
FT91	1.554	0.942	0.847	5.527	0.900	0.825	4.655	5.890
CR	5.376	3.272	3.033	18.053	3.010	2.425	15.910	19.890
FT92	21.726	12.287	11.603	65.680	8.735	7.390	57.815	71.050
ZIFF	21.592	12.815	11.888	67.120	9.070	8.020	58.790	72.340
FT93	23.398	13.608	12.675	71.233	10.040	9.345	62.565	77.860
FT94	23.802	13.837	13.013	75.925	10.845	10.020	62.795	80.370
AP	32.184	17.623	16.923	88.823	15.990	13.200	81.875	103.010
ALL_FT	75.934	42.055	39.970	214.180	36.295	30.430	189.905	235.370
ALL	161.448	87.488	81.893	394.067	62.485	56.510	328.240	432.390

Table 11.5: Comparison against *gzip*, *bzip2*, and arithmetic technique.

ratio than "gzip -f".

Both Dynamic ETDC and Dymamic SCDC achieve also better compression ratio than "gzip -b" and they are much faster. However, among all the dynamic techniques used, *gzip* is clearly the fastest method in decompression. This fact justifies its interest for some specific applications. Notice that "gzip -f" is slower than "gzip -b" in decompression. This is because the decompression has to be performed over a smaller compressed file (since "gzip -best" compresses more than "gzip -fast").

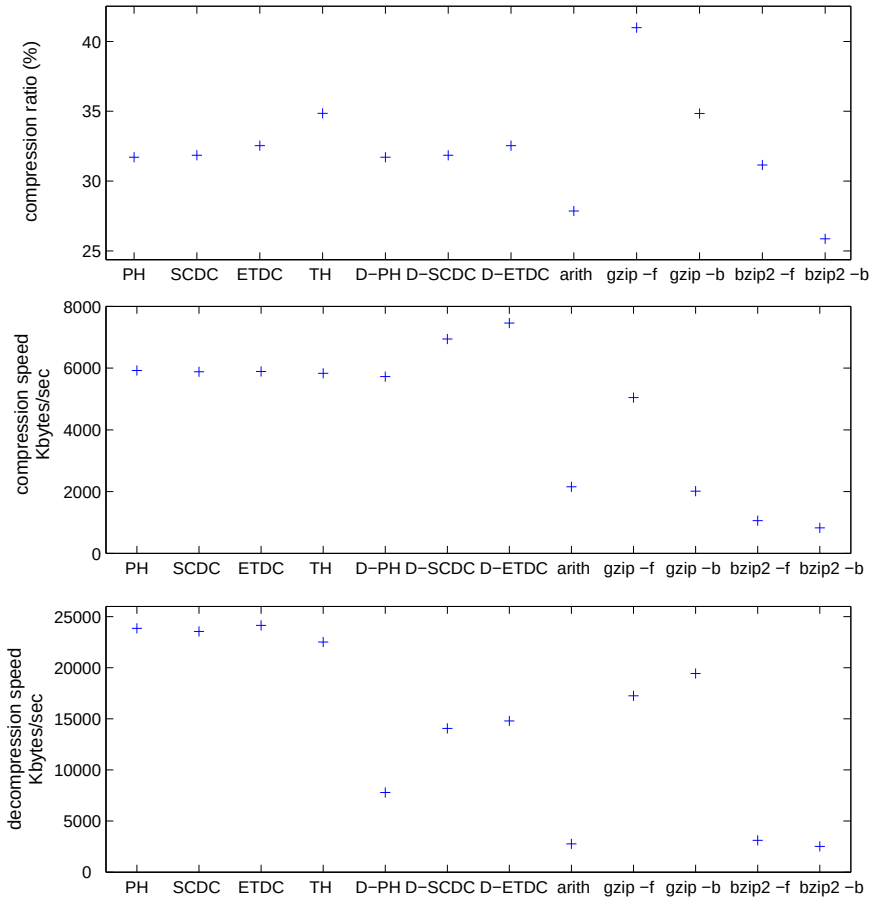
Regarding decompression speed, Dynamic PH decompresses about 8 Mbytes per second, while Dynamic SCDC and Dynamic ETDC achieves about 14 and 15 respectively. Therefore, Dynamic SCDC decompresses about 40% faster than Dynamic PH, and Dynamic ETDC is about 45% faster in decompression.

Hence, Dynamic ETDC is, in general, faster and always compresses much better than *gzip*, and it is by far faster than *bzip2*. Dynamic SCDC is also a good alternative to *gzip*. It is faster than "gzip -f" in compression, and its compression ratio is much better. Regarding Dynamic PH, it is also a well-balanced technique. It is about 20%-25% slower than Dynamic ETDC (17% with respect to Dynamic SCDC) but its compression ratio is a little better.

With respect to the arithmetic compressor used (which also uses a word-based approach, but it is bit-oriented rather than byte-oriented), it displays the advantages in compression ratio of bit-oriented word-based techniques. However, it can be seen that compression speed is about half of Dynamic PH and decompression is about 3 times slower than the byte-oriented techniques presented.

To sum up, we have presented empirical results comparing the compression ratio, as well as the compression and decompression speed obtained by each compression technique over different corpora. In compression ratio, *bzip2* (followed by the bit-oriented arithmetic code) achieves the best results, and *gzip* is clearly the fastest dynamic decompression technique. On the other hand, *Dynamic ETDC*, *Dynamic SCDC*, and *gzip* are the fastest compressors among all the dynamic techniques compared.

Figure 11.7 summarizes both compression ratio, compression speed, and decompression speed for the distinct techniques working over the FT_ALL corpus. Notice that we have also included the semi-static techniques in that figure.



METHOD	PH	SCDC	ETDC	TH
Comp ratio	31.696	31.839	32.527	34.850
Comp speed	5,922	5,882	5,888	5,830
Decomp speed	23,856	23,552	**24,146**	22,510

METHOD	D-PH	D-SCDC	D-ETDC	arith	gzip -f	gzip -b	bzip2 -f	bzip2 -b
Comp ratio	31.710	31.849	32.537	27.852	40.988	34.845	31.152	**25.865**
Comp speed	5,724	6,944	**7,459**	2,157	5,045	2,015	1,059	824
Decomp speed	7,791	14,067	14,800	2,762	17,249	19,440	3,115	2,513

Figure 11.7: Summarized empirical results for the FT_ALL corpus.

11.6 Summary

In this chapter, Dynamic ETDC was extended to maintain the s and c parameters optimal at any moment of the dynamic encoding/decoding processes. The result is a dynamic SCDC compressor.

Dynamic SCDC code achieves compression ratios very close to those obtained by the Huffman-based dynamic technique presented in Chapter 9. Moreover, Dynamic SCDC is faster in both compression and decompression and it is easier to build.

With respect to Dynamic ETDC, Dynamic SCDC gets a little worse in compression and decompression speed. However, compression ratios are better and very close to Huffman values.

Empirical results comparing our three contributions in dynamic text compression techniques, against well-known compressors have been presented. As a result, we can conclude that we have obtained good tradeoffs in adaptive natural language text compressors, with about 30%-35% compression ratio, more than 5-6 Mbytes per second in compression speed and about 8-15 Mbytes per second in decompression speed.

Results show that the dynamic compression techniques developed in this work are competitive alternatives, in both compression ratio and speed, to the most spread and well known techniques.

12

Conclusions and Future Work

The main contribution of this work is a completely new family of statistical compressors called *dense* compressors that have been designed and developed in this thesis. Observe that four of the five compression techniques developed (ETDC, (s, c)-DC, Dynamic ETDC and Dynamic (s, c)-DC) have in common that they take advantage of a *dense* coding scheme, that is, they use all the possible bit combinations that can be built to write target symbols. This feature provides them with a powerful compression capacity that overcomes any other statistical code, semi-static or dynamic respectively.

Note that any other statistical code has to reserve some of those bit combinations to obtain a prefix code. Hence, for example, ETDC compresses strictly better than Tagged Huffman Code, precisely due to the property of exploiting all the possible bit combinations, taking into account that only the bits that are not used as tag (7 in the implementations developed for both ETDC and Tagged Huffman Code) are used for encoding.

As a consequence of this basic property of being dense codes, other very interesting features arise and, therefore, they are common characteristics for all the codes in the *dense* family.

One of these characteristics of the dense codes is the simplicity of their code generation. Obviously, this second feature is a consequence of the first one. Due to the use of all bit combinations, it is possible to generate the codewords in an almost

sequential procedure or, alternatively, with a simple operation similar to a change of base (Sections 5.2 and 6.2). Therefore, the implementation of the statistical encoding process that maps shorter codewords to the most frequent symbols, that is, the implementation of the *codebook*, can be easily programmed and achieves good running performance. The same can be applied to the decoder procedure, where a simple computation (similar to a numerical base change) gives the word rank i from a codeword. Then, the rank i can be used to access the word i in the vocabulary.

Another interesting peculiarity of these compressors, also arising from the basic characteristic of using *dense* codes, is that the frequency of the source symbols is not considered during the assignment of codewords. To encode a word, dense codes only use the word (source symbol) rank, but not its actual frequency. In this way the encoding scheme is more stable than in Huffman, where a change in a word frequency, even if it does not change its rank, could modify the shape of the tree. Notice that if such a change occurs using ETDC, the codeword assignment does not change. In the case of using (s,c)-DC, changes of frequencies that do not affect the word rank, could only produce modifications of the s value in extreme cases. This stability is an interesting characteristic that we are currently exploring in order to develop new *dense* compressors that will be presented as Future Work.

We have to emphasize that using these encoding schemes, which consider the word rank instead of the frequency of each source symbol, does not imply a loss of compression capacity since they are dense.

Although we have already seen that these codes are simpler and compress better than previous methods, they maintain one of the most important characteristics of some recent compression techniques, it is possible to search directly in the compressed version of the text using any pattern matching algorithm from the Boyer-Moore family.

The dynamic versions, presented in Chapters 10 and 11, do not maintain the property of being searchable, but we have to keep in mind that these methods were developed without having this property as a goal. However, we are currently working in a variation of the D-ETDC to give it more codeword stability [BFNP05], in order to provide D-ETDC with direct search capability. We present these ideas in the Future Work section.

After this general overview of the family of codes developed in this work, in the next section we present in detail the main achievements of these codes, which represent the contributions of this Thesis.

12.1 Main contributions

The first objective, the development of new codes well-suited for its integration into text retrieval systems, is faced in the first part of the thesis. This goal was accomplished by the development of two semi-static techniques: the ETDC and a generalization of it called (s, c)-Dense Code ((s, c)-DC). (s, c)-DC adapts its parameters s and c to the word frequency distribution in the text being compressed, and improves the compression ratio achieved by ETDC about 0.7 percentage points. (s, c)-DC is faster and almost obtains the same compression ratio obtained by the optimal Plain Huffman (Section 4.2). ETDC is even faster than (s, c)-DC at the expenses of losing around 1 percentage point with respect to Plain Huffman. Therefore, both ETDC and (s, c)-DC obtain compression ratios close to Plain Huffman, and maintain the good features of other similar compression schemes such as the Tagged Huffman code:

- They are word-based techniques.
- They produce a prefix code encoding.
- They enable decompression of random portions of a compressed text, by using a tag condition that permits to distinguish codes inside the compressed text.
- They enable direct search in the compressed text, and consequently they improve searches, in such a way that searching (s, c)-DC or ETDC can be up to 8 times faster, for certain queries, than searching the uncompressed text, as it happened with Tagged Huffman [MNZBY00].

These new codes incorporate new advantages over the existing Huffman-based ones:

- Compression ratios are better than in Tagged Huffman.
- Encoding and decoding processes are faster and simpler than those of Huffman-based methods.

To sum up, two almost optimal, fast and simple compression techniques were developed.

The second part of the thesis covers the development of the three dynamic techniques proposed: the Dynamic word-based byte-oriented Huffman code, the

Dynamic ETDC and the dynamic version of (s, c)-DC. These techniques are based on the semi-static word-based Plain Huffman Code, ETDC, and (s, c)-DC respectively.

These dynamic techniques become well-suited for both file compression and, specially, real-time transmission. The most interesting features of those three compression methods are the following ones:

- They join the real-time capabilities of the previous adaptive character-oriented Huffman-based techniques, with the good compression ratios achieved by the statistical semi-static word-based compression techniques. In fact, the loss of compression is negligible with respect to the semi-static versions of the codes.
- Being byte-oriented, compression and decompression processes are very fast. In compression, the Dynamic ETDC and the Dynamic (s, c)-DC are faster than the well-known *gzip* technique. Moreover, even being slower than *gzip* in decompression, the decompression speed is very competitive.

12.2 Future work

We are convinced that we have not used up all the capabilities of the ETDC, the (s, c)-DC and the dynamic versions of those codes. On the contrary, we consider that the application of the dense codes to new contexts and needs may lead us to the development of new compressors of this family. In the short term we plan to tackle three research lines:

1. **Searchable dynamic dense codes**. The main idea of these codes was sketched in [BFNP05]. The main goal of that work is to obtain a dynamic version of ETDC with direct search capability. The codeword assigned to a word w_i remains stable while w_i does not change to another position where it should be encoded with a different number of bytes.

   Remember that using ETDC, the rank of words in the vocabulary permits the identification of the boundaries of the zones where all words are encoded with codewords of the same size. For example, those words that are placed in the vocabulary between positions 0 and 127 belong to the zone where all words are encoded with codewords of 1 byte, therefore 127 is the upper boundary of this zone, since words in positions 128, 129,... are encoded with codewords of 2 bytes. Our new version of the D-ETDC does not change the codeword

assigned to a word w_i as long as (after a change in its frequency) its rank position in the vocabulary does not cross the boundaries of its current zone.

Each time a word crosses the boundaries of that zone, the sender/encoder has to use a scape codeword to indicate to the receiver/decompressor that two words are exchanging their positions, and therefore their codewords have to be exchanged as well. Since the number of changes of position is small, the decompressor/receiver does not have to perform much work to keep its vocabulary synchronized with the encoder/sender, and therefore, decompression is much faster than in D-ETDC.

This dynamic compressor takes advantage of one of the properties of the dense codes, the stability of the encoding schema due to the fact that codewords are obtained from the word rank, instead of from the actual frequency of each source symbol. As a result, we obtain a code that allows direct search, although it is a dynamic code. This dynamic code could be useful in a new set of scenarios. For example, the receiver would not have to decode the message, it might only have to classify it by searching a word or a set of words (inside the compressed message) and then, to store it compressed in the appropriate directory. This code would be also suitable in mobile computation because the decoder is very simple (even simpler that the one described in this thesis).

2. **Variable to variable codes**. This subject is a relatively new research field [SSed]. The idea behind this codes is the use of source symbols of variable length, in our case, source symbols composed by a variable number of words and separators. Obviously, at the same time, the code should assign variable length codewords to such source symbols in order to obtain good compression.

   We think that dense codes will lead us to new developments that have not been even foreseen with other statistical variable to variable techniques.

3. **Compression of growing collections.** We are working in the design of a dense compressor to deal with growing collections avoiding the need of recompression. All semi-static codes have an important drawback, the whole corpus being compressed has to be available before compression process starts.

   Dynamic codes do not have this limitation, but these codes do not permit performing direct search in the compressed text nor decompressing arbitrary portions of the text, since the process needs to start the decompression from the beginning of the corpus.

   Digital libraries, a research field that is becoming more and more attractive, has to face this problem, specially if the digital library in embedded in the

Web. Compression allows to save disk space and transmission time, but the digital library corpus, in most cases, grows continuously, hampering the use of the semi-static methods. On the other hand, digital libraries should include information retrieval services, therefore compression techniques might allow direct searches inside the compressed text and the decompression of arbitrary portions of the text. Hence dynamic codes are not suitable.

Obviously, we always have the chance of compressing each new addition isolated, that is, with an individual vocabulary for each new portion of the corpus that is added to the whole corpus. However, this implies that many words will be repeated in many different vocabularies. It would be desirable that the complete corpus would have a unique vocabulary, avoiding the need of encoding the same word with each new addition.

Currently, we are working on the development of a compressor that is specially adapted to digital libraries [BFNPed]. When adding more text to an existing text collection, new words usually appear. Moreover, since there are words such that their frequency is increased, a shorter codeword should be assigned to them to avoid losing compression. Instead of giving a shorter codeword, what implies reencoding the previous text in the digital libray, our new strategy is based on joining two words into one, and giving to it a (usually larger) unused codeword. Notice that, since two words are encoded with a unique codeword, not only compression does not worsen but it is even improved if the new joined-word appears several times.

The simplicity of the encoding process of the dense codes facilitates the generation of codewords for the new words that appear with the new additions and also the generation of codewords that represent more than one original word. We already have preliminary experimental results that show a promising research line.

Appendix A

Publications and Other Research Results Related to the Thesis

A.1 Publications

A.1.1 International Conferences

- N. Brisaboa, A. Fariña, G. Navarro, and J. Paramá. Efficiently decodable and searchable natural language adaptive compression. In *Proceedings of the 28th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR-2005)*, Bahia-Brazil, August 15–19 2005. ACM Press. To appear.
- N. Brisaboa, A. Fariña, G. Navarro, and J. Paramá. Simple, fast, and efficient natural language adaptive compression. In *Proceedings of the 11th International Symposium on String Processing and Information Retrieval (SPIRE 2004)*, LNCS 3246, pages 230-241, Padova, Italy, 2004. Springer-Verlag.
- M. Marin C. Bonacic, A. Fariña, and N. Brisaboa. Compressing Distributed Text in Parallel with (s, c)-Dense Codes. In *Proceedings of the 24th International Conference of the Chilean Computer Science Society (SCCC)*, pages 93-98, IEEE CS Press, 2004.

- A. Fariña, N. Brisaboa, C. Paris, and J. Paramá. Fast and flexible compression for web search engines. In *Proceedings of the 1st international workshop on Views on Designing Complex Architectures (VODCA 2004)*, Bertinoro, Italy, 11-12 Sept. To appear in Electronic Notes in Theoretical Computer Science. Elsevier, 2004.

- N.R. Brisaboa, A. Fariña, G. Navarro, and M.F. Esteller. (s,c)-dense coding: An optimized compression code for natural language text databases. In *Proceedings of the 10^{th} International Symposium on String Processing and Information Retrieval (SPIRE 2003)*, LNCS 2857, pages 122–136, Manaos, Brazil, 2003. Springer-Verlag.

A.1.2 National Conferences

- E.V. Fontenla, A.S. Places, N. Brisaboa, A. Fariña, and J. Paramá. Recuperación de textos en la biblioteca virtual galega. In *Actas de las IX Jornadas de Ingeniería del Software y Bases de Datos*, pages 87–98, 2004.

- N. Brisaboa, A. Fariña, G. Navarro, E. Iglesias, J. Paramá, and M. Esteller. Codificación (s, c)-densa: optimizando la compresión de texto en lenguaje natural. In *Actas de las VIII Jornadas de Ingeniería del Software y Bases de Datos*, pages 737-746, 2003.

- E. Iglesias, N. Brisaboa, J. Paramá, A. Fariña, G. Navarro, and M. Esteller. Usando técnicas de compresión de textos en bibliotecas digitales. In *Actas de las IV Jornadas de Bibliotecas Digitales*, pages 39-48, 2003.

A.1.3 Journals and Book Chapters

- E.V. Fontenla, A.S. Places, N. Brisaboa, A. Fariña, and J. Paramá. Recuperación de textos en la biblioteca virtual galega. *IEEE América Latina*, 3(1), 2005.

- N. Brisaboa, A. Fariña, G. Navarro, and E. Iglesias. Compresión de textos en Bases de Datos Digitales. In Ingeniería del Software en la Década del 2000, pages 169-180, *AECI*, 2003. Spain.

A.2 Submitted papers

A.2.1 International Journals

- N. Brisaboa, A. Fariña, G. Navarro, and J. Paramá. Semi-static dense compressors. Manuscript to submit to *Information Retrieval*. Springer.
- N. Brisaboa, A. Fariña, G. Navarro, and J. Paramá. Dynamic dense compressors. Manuscript to submit to *Information Processing and Management*. Elsevier.

A.2.2 International Conferences

- N. Brisaboa, A. Fariña, G. Navarro, and J. Paramá. Compressing dynamic text collections via phrase-based coding. In *Proceedings of the 9^{th} European Conference on Research and Advanced Technology for Digital Libraries (ECDL 2005)*, September 18-23 2005. Submitted.

A.3 Research Stays

- *August* 26^{th}*, 2003 - October* 6^{th}*, 2003.* Research stay at the Universidad de Chile in Santiago, Chile, under the supervision of Prof. Dr. Gonzalo Navarro.

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